EC6012 Lecture 2
Numerical Example

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1 Lecture Outline

First, some housekeeping.

Today, we'll go back over what we learned last time, and extend the basic SIM model to allow for the basic Keynesian multiplier by hand. You'll see some simulations of the model to the steady state, and see how it accommodates expectations and long run values. We'll close with a discussion of your papers.

Notation

Symbol	Meaning
G	Pure government expenditures in nominal terms
Y	National Income in Nominal Terms
C	Consumption of goods supply by households, in nominal terms
T	Taxes
θ	Personal Income Tax Rate
YD	Disposable Income of Households
α_1	Propensity to consume out of regular (present) income
α_2	Propensity to consume out of past wealth
ΔH_s	Change in cash money supplied by the central bank
ΔH_h	Cash money held by households
H, H_{-1}	High Powered cash money today, and yesterday ($_{-1}$)

2 Generating the standard Keynesian Multiplier

Review equations 3.1–3.12 in chapter 3. They derive a simple stock-flow consistency model which we can use to simulate the effects of a simply Keynesian multiplier.

With a tax rate of 20%, and assuming the parameters of the consumption function are $\alpha_1 = 0.6, \alpha_2 = 0.4$, the baseline case is zero economic activity. So, you should expect to see nothing in any of the rows in the first period.

Now the government is 'born' in some sense, and they start ordering things from the firms, who employ the households to produce the stuff being ordered, and off the economy goes.

The government initially orders (or *injects*) \$20 worth of stuff, which circulates around the system such that the households get the 20 in wages for producing the stuff, then they must pay taxes of 20% on this. This money just gets destroyed in this example once it goes back to the government.

The households go off and buy things from each other to the value of \$16 in this period. The model continues this process ad infinitum, with the resultant being that the initial injection of \$20 causes ripples throughout the economy and we get a multiple effect on the economy, hence the term "multiplier".

Our system of equations looks like this:

$$
\begin{aligned}
G & & (1)\\
Y &= G + C & (2)\\
T &= \theta \times Y & (3)\\
YD &= Y - T & (4)\\
C &= \alpha_1 \times YD + \alpha_2 \times H_1 & (5)\\
\Delta H_s &= G - T & (6)\\
\delta H_h &= YD - C & (7)\\
H &= \Delta H + H_{-1} & (8)
\end{aligned}
$$

If we start by solving the model for Y, everything will become clear. Thus $Y = G + C$ and $T = \theta Y$, and by substituting in for T and factoring, we get

$$
\begin{aligned}
YD &= Y - T & (9)\\
&= Y \times (1 - \theta). & (10)
\end{aligned}
$$

By similar logic, $C = \alpha_1 \times YD + \alpha_2 \times H_{-1}$.

Since, in period 2, $H_{-1} = 0$, we can say that $C = \alpha_1 \times Y(1-\theta)$. Substitute this into $Y = G + C$ and we get

$$
\begin{aligned}
Y &= G + \alpha_1 Y(1-\theta), & (11)\\
Y - \alpha_1(Y)(1-\theta)) &= G, & (12)\\
Y[1 - \alpha_1 \times (1-\theta)] &= G, & (13)\\
Y &= \frac{G}{1 - \alpha_1 + \alpha_1\theta} & (14)
\end{aligned}
$$

We have numbers for $\alpha_1, G[\text{Period1}]$, and θ—0.6, 20, and 0.2. Plugging these into equation (14), we can calculate Y for period 2. We obtain

$$Y = \frac{20}{1 - 0.6 + 0.6 \times 0.2} = 38.462 \simeq 38.5.$$

As soon as you have solved for Y, you can fill in all the remaining numbers in column 2 including ΔH and therefore H. You now have all the material you need to solve for Y in period 3 ($H_{-1} = 12.3$) and the whole column in period 3. And so on.

The system reaches a steady state when $\Delta H = 0$ and hence $YD = C$.

3 Exercise 1

Do this now. Fill in all the values for column 2 of table 3.4 and show your workings. Ask me if you get stuck.

Period	**1**	**2**	**3**	∞
G	0	20	20	20
$Y = G + C$	0	38.5	47.9	100
$T = \theta \cdot Y$	0	7.7	9.6	20
$YD = Y - T$	0	30.8	38.3	80
$C = \alpha_1 \cdot YD + \alpha_2 \cdot H_{-1}$	0	18.5	27.9	80
$\Delta H_s = G - T$	0	12.3	10.4	0
$\Delta H_h = YD - C$	0	12.3	10.4	0
$H = \Delta H + H_{-1}$	0	12.3	22.7	80

Figure 1: Table 3.4 of Godley/Lavoie.

3.1 Exercise 2

What happens to this model if θ changes from 20% to 30%? Work out the first period and then give and economic explanation for the figures you see.

4 Steady State

We saw in the last lecture that the Steady State is a position of dynamic balance, where all the variables in the economy are changing at the same rate, that is, their *levels* or ratio values are constant over time. Once we have this situation in an economy, we can start asking questions like what happens when the government finds itself in this situation? The government must find itself with levels of government expenditures equal to some optimal level of tax takings, so $G = T^*$, and these tax takings will be optimal (no surplus or deficit) because they are levied on the output of workers, $\theta \times W \times N^*$. Because the sum of the output of workers is the output of our primitive economy, it must be the case that the tax rate is optimally leveled on output, so $\theta \times W \times N^* = \theta \times Y$. Connecting all these relationships (because they are equivalent in the long run), we have that

$$Y^* = \frac{G}{\theta}. \tag{15}$$

Equation 15 gives us the stationary state flow of aggregate income, Y^*.
Now look at the simulation in Eviews.

4.1 Exercise 3

What do you think will happen to the steady state value(s) of output when θ changes? Why does this happen? Post the answers on your blogs by next Monday.

5 Stock-Flow Consistency in the Steady State

We have already made sure that stocks and flows balance through our accounting procedures. In the steady state, the extra information we are allowed to use because of this balance helps us redefine the consumption function in terms of optimal stocks and flows. In the steady state, the level of consumption is not changing. Thus after a little derivation,

$$\begin{aligned} C &= YD - \Delta H_h && (16) \\ &= \alpha_1 \times YD + \alpha_2 \times H_{h-1} && (17) \\ \delta H_h &= (1-\alpha_1) \times YD - \alpha_2 \times H_{h-1} && (18) \\ \Delta H_h &= \alpha_2 \times (\frac{1-\alpha_1}{\alpha_2} \times YD - H_{h-1}) && (19) \end{aligned}$$

Equation (19) says that wealth is being accumulated at a persistent rate, which is dependent on α_2, the proportion of past wealth. The fraction $\frac{1-\alpha_1}{\alpha_2}$ specifies the desired level of wealths by households, or what Godley calls *the stock-flow norm* (pg. 75). This relationship gives a target level of wealth to attain over the lifetime of the household.

6 Expectations and Dynamics

Expectations are a large part of modern macroeconomics. This simple model incorporates the notion of expectation formation by reducing the level of information the households have in the system from perfect to less-than perfect, and including another equation describing *how* households would go about forming their expectations. The consumption function gets redone to look like this:

$$C_d = \alpha_1 \times YD^e + \alpha_2 \times H_{h-1}. \qquad (20)$$

Here YD^e is the expected value of disposable income. Assume that households have some estimate of their consumption out of disposable income in the current period, we can define H_d, the demand for money by households, and describe this money demand via

$$\Delta H_d = H_d - H_{h-1} = YD^e - C_d. \qquad (21)$$

which gives

$$H_h - H_d = YD - YD^e. \qquad (22)$$

Equation 22 shows that if realised income is above expected income, then households will save more money by holding larger real balances. The opposite is also true. Here we have a *realistic* description of the capitalist system of accumulation and money 'buffering' that is not found in mainstream macroeconomics.

	1. Households	2. Production	3. Government	Σ
1. Consumption	$-C_d$	$+C_s$		0
2. Govt. expenditures		$+G_s$	$-G_d$	0
3. [Output]		$[Y]$		
4. Factor income (wages)	$+W \cdot N_s^e$	$-W \cdot N_d$		$W \cdot N_s^e - W \cdot N_d$
5. Taxes	$-T_s^e$		$+T_d$	$T_d - T_s^e$
6. Change in the stock of money	$-\Delta H_d$		$+\Delta H_s$	$\Delta H_s - \Delta H_d$
Σ	0	0	0	0

Figure 2: SIM with Expectations.

6.1 Dynamics

What happens when the perfect foresight model is out of equilibrium? There must be some rule or dynamic equation to bring the system back into equilibrium. This is achieved by looking at difference equation versions of the output relationship we studied above. First look at how we solved for Y in the numerical example above. We looked at solutions for Y in each period, depending on the value of Y that came before it. The true value of Y when we don't specify which period to solve for is

$$Y = \frac{G + \alpha_2 \times H_1}{1 - \alpha_1 \times (1 - \theta)}. \tag{23}$$

Then the household's demand for money is

$$H_h = (1 - \alpha_1) \times (1 - \theta) \times Y + (1 - \alpha_2) \times H_{-1}. \tag{24}$$

Each time we sub in the value of Y obtained in equation (23), the current value of wealth pops out, because we know everything else in the equation. The out of equilibrium relationship of consumption and wealth when α_1 changes from 0.6 to 0.7 is given in figure 3.8 and I'll show it in class. This dynamic process is stable as section 3.8 of GL shows.