Provision of Public Good in a Federal Economy:
The Role of Party Politics

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Abstract: We analyze the role of political parties in provision of public goods within a federal economy. The public goods are federally funded but locally produced (with costly efforts), and have interjurisdictional spillover effects. The direction and magnitude of fund flow, which ultimately determine the local provision of public goods, are influenced by the re-election probability of the parties in power at the federal and provincial levels. The prevailing wisdom is that provincial governments, which are ruled by the same political party as that ruling at the federal level, enjoy a higher level of the public good compared to provincial governments which are ruled by a different political party. We demonstrate that there exist incentive effect of federal transfer that complements such partisan effects.

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1 Introduction

In this paper, we attempt the following set of questions in the context of a federal economy. How does a politically motivated federal authority dispense federal discretionary transfer among provinces? How does this affect the provincial incentive to provide public good with local efforts? What is the ultimate effect on provincial effort? How does politics affect the distribution of public good across provinces? Does it necessarily reduce federal welfare?

These questions arise because in a federal country, provinces rely on federal transfer to provide a local public good. Such reliance, which is implied by two imbalances, is an endemic feature of federal countries. Provincial expenditures may exceed provincial revenues (vertical imbalance), or the provinces may be unable to provide comparable levels of same public good at comparable prices (horizontal imbalance). These considerations necessitate fund flow from the federal authority to provinces. The bulk of such transfers are formulaic in nature. However, the federal authority often takes recourse to discretionary ex post transfers, such that, ostensibly, a sense of horizontal equity prevails within the federation. This discretionary nature of such transfers may betray a political element, such that provinces with otherwise similar performances may end up with different levels of transfers and public good. This feature is well documented in literature. A few studies (See, for example, Porto and Sanguinetti 2001; Khemani 2003; as well as Sollé-Ollé and Sorribas-Navaro 2006) find the evidence of partisan transfers (such that jurisdictions/provinces that share the same political

\footnote{Transfers that arrive after observing the provincial effort in public good production.}
identity with the upper level governments receive more transfers). On the other hand, Dasgupta et al. (2003) or Johansson (2003) found the evidence that central transfer are distributed to maximise re-election probability, not necessarily to reward loyal provinces.

It is well recognized (see e.g. Oates, 1979) that federal transfers do alter provincial behaviour at the margin. Moreover, if the federal government cannot commit to a level of transfer (such as the case of ex post transfers), then provinces perceive the ex post federal transfer and local efforts to be substitutes. Strategic provinces impose inefficiently low amount of tax,\(^3\) reduce public good provision, invest in ‘wrong’ type of public good or simply exceed their budget in anticipation to the ultimate bail out by the federal authority. Examples include the literature on soft budget constraint (a good survey is provided by Kornai et al, 2003). In Köthenbürger (2006; 2007), the presence of strategic behavior by provinces, equalization transfers generate such inefficiency. There is some recent empirical evidence in Zhuravskaya (2000). The author shows that, the Russian decentralization process does not provide adequate fiscal incentives to the local governments to increase local revenue. A shortcoming of this strand of literature is not taking federal discrimination due to political factors into account.

Political discrimination among provinces have been discussed in a series of papers by Dixit and Londregan (1995; 1998a; 1998b, hereafter DL). In their analysis, voters are differentiated along two dimensions: ideology and consumption.\(^4\) A party can attract a voter of rival ideology if the con-

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\(^3\)Smart (1998) notices that provinces may also impose higher tax to reduce tax base in anticipation of higher equalization transfer.

\(^4\)Consumption is directly linked with transfer.
sumption (i.e. transfer) package it offers is high enough to offset ideological opposition. More funds flow to provinces where the marginal response to increase in consumption is highest: so these are the politically favorite ones. A key insight that emerges from DL analysis is that a province in which the voters are too loyal (or disloyal) in terms of political ideology, receives less federal transfer vis-à-vis other provinces. However, these papers are concerned with pre-election politics where parties make a binding commitment towards post election transfer. There is no treatment of ex post transfer and their incentive effects.

This paper attempts a synthesis of the two above-mentioned strands in the literature on federal transfers. Specifically, we examine how ex-post federal transfers, which are motivated by political expediency, affect provincial behaviour. We consider a situation where each province produces a public good with its own revenue raising effort supplemented by federal transfer. The public good has spillover benefits across provinces. Federal transfers are lump sum and follow a net equalization program: thus the transfer to one province is balanced by an equal tax on the other. Therefore, the total public good production at the federal level remains the sum of provincial efforts. In this world, federal transfers are conditioned by provincial behavior.

To simplify the analysis, we assume that there exist two parties and two provinces within the federation. Each province is governed by a different

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5 They employ the electoral competition model of Lindebeck and Weibull (1987). For a general discussion and further applications, see Persson and Tabellini (2000). The papers of Dasgupta et al. and Johansson, op. cit., provide some empirical validity for DL approach.

6 For excellent surveys of normative analysis of equalization, see Boadway (2001; 2004).
party (incumbent) while the other party is in opposition (challenger). A representative voter’s utility increases with the public good but decreases with efforts (which are costly). He compares the net provincial utility levels as an indicator of the (in)competence of the incumbent party in the province vis-à-vis the challenger. Vote shares for the parties become a function of the difference between provincial utilities. Federal welfare is the sum of provincial utilities. Taking the voter behaviour into account, federal transfers are designed in such a way that a sum of federal welfare and vote share is maximized. Thus, the federal government is partly benevolent.\footnote{It may be benevolent due to constitutional constraints.} We make the simplifying assumption that provincial governments choose effort levels to maximize provincial welfare, ignoring the spillover effect ("naïve" provincial governments).\footnote{The basic result go through if the provinces are vote share maximising as well. The only qualitative difference that occurs is that there is shrinkage in public good production.}

Using such a framework, the current paper systematically analyzes the effect of federal transfers on provincial revenue raising efforts, local and federal public good production as well as local and federal welfare. The main results that emerge from our analysis are the following. We show that, \textit{ceteris paribus}, a politically motivated federal transfer induces higher public good consumption (through higher transfer) to the favorite province than a non political regime. For a broad class of utility functions, local incentive as well as the effort level in the favorite province falls and that in the other province increases. Therefore, the effect on public good in favorite province remains ambiguous. We show an example where it actually falls. On the other hand, total provincial revenue may also rise and fall. Accordingly, the
The total amount of public good produced within the economy (sum of provincial revenues) also falls in the case when the increase in effort from the non-favorite province is not enough to wipe out the reduction in effort from the favorite province. Turning on welfare, we show that political transfers unambiguously increase the welfare level in the favorite province. Since it discourages the local effort in the first province, and increases that in the second province, politically motivated transfers may enhance federal welfare (thus being more efficient, compared to non-political transfer) in the case where the marginal cost of revenue is higher in the first province. We provide an example of this.

The presentation is divided into following sections. In section 2, the basic model is described. Section 3 contains discussions on non-commitment. In section 4, the effect of political transfer on provincial and federal public good consumption is discussed, while section 5 concludes.

2 The Model

There exist two provinces, denoted by 1 and 2. A representative consumer in either province derives utility from the public good \( P_i \) and private wealth \( w_i \). The utility function is depicted as

\[
U_i(w_i, P_i) = u_i(P_i) + g_i(w_i), \quad i = 1, 2
\]  

(2.1)

The following usual assumptions are made on \( u_i \) and \( g_i \):

\[ u_i' > 0 > u_i'' \]

\[ g_i' > 0 > g_i'' \]

The total public good consumed in province \( i \) includes ‘own’ production \( p_i \) and spillover from the neighboring province. To begin with, we assume
that spillover is symmetric. The total public good in province $i$ is thus given by

$$P_i = p_i + \beta p_j \quad \text{for } i \neq j \quad \text{where } i, j = 1, 2$$

Here, $\beta \in [0, 1)$ is the spillover effect.

A public good produced within the province is financed by locally procured taxes/or efforts ($\theta_i$) and transfer from the federal government:

$$p_i = B_i + \theta_i + T_i$$

$B_i > 0$ is the status quo public good produced in province $i$ independent of the tax/transfer scheme and $T_i$ is the federal transfer. Federal transfer might be negative (for equalization purpose). We assume that $B_i$ is large enough such that the stock of public good does not fall below zero.

Wealth is consumer income ($Y_i$) less taxes. We assume that income is given. So we have the following:

$$w_i = y_i - \theta_i$$

We assume that $Y_i > 0$ and $\theta_i \in [0, Y_i)$. The magnitude of marginal cost of effort is thus given by $g_i'(y_i - \theta_i)$. Ceteris paribus, higher $y_i$ implies lower marginal cost. Thus a wealthy province can raise the more easily.

The federal government is constrained by a fixed budget due to intergovernmental transfer. We assume that transfer to one province is financed by taxing the other province. Therefore, we can write,

$$T_1 + T_2 = 0$$
Comment: Thus, the federal government embarks upon a net equalization scheme. The net transfer to a province can be negative or positive, and the budget must balance.

The benchmark conditions are associated with a benevolent and nonpolitical unitary government that can choose both \( T_i \) and \( \theta_i \) to maximize the sum of utilities as represented in (2.1). The associated first best conditions are:

\[
\begin{align*}
  u_i' &= u_j' \quad \text{(2.2)} \\
  u_i' + \beta u_j' &= g_i' \quad \text{(2.3)}
\end{align*}
\]

Thus the federal government equates the marginal utility across provinces as shown by equation (2.2) and impose a tax on provinces such that a modified Samuelsonian condition is satisfied as shown by equation (2.3). These two imply, inter alia, that \( g_1' = g_2' \). Therefore, the marginal costs of raising public fund (MCF) are equalized across provinces.

Suppose \( g_1 = g_2 = g \) and \( y_1 = y_2 = y \). Then the first best case implies that \( \theta_1 = \theta_2 \) such that the federal authority chooses equal effort from both provinces.

2.1 Political Framework

There are two political parties. We assume that party L is in power in centre and province 1, while party R governs province 2. In each province, as well as in the centre, the party that is not in power is called the opposition or the
challenger. There is an election at the federal level. Each province chooses one representative to the centre (without disturbing the composition of the provincial governments).\footnote{On another note, we can also think as if there is a local election and the federal government cares for the local election.} The party which is in the centre determines the transfer in such a way that it’s own vote share is maximized from both provinces.\footnote{It must be recognised that the regional governments can be strategic as well. This modification is discussed later.}

To abstract away from the interaction between voters and politicians, and to focus more on the action of politicians at different levels of government, we make the following simplifying assumption:

**Assumption V**: Voters are non strategic in nature and they commit to a voting rule.

Voters in a province care for relative utility. This can be rationalized through status-seeking motive. If in the neighboring province, utilities are relatively higher, the vote share of the incumbent in a province goes down. On the other hand, if they observe that the utilities are higher in their province, the vote share of the provincial incumbent goes up. To capture such behaviour in the simplest possible framework, we propose the following. In province 1, a voter \(i\) will vote for the incumbent if and only if \(U_1 \geq U_2 + \delta_{1i}\). Here \(\delta_{1i}\) is a random variable that captures the voter heterogeneity for voter \(i\) in province 1. It can be positive or negative.\footnote{In a formal model of yardstick competition (e.g. Besley and Case 1995), relative performance evaluation serves as a sorting mechanism between good and bad politician in presence of adverse selection and moral hazard. Our model abstracts away from such}
Here, the voter who is indifferent between choosing the incumbent or challenger is situated at $\delta_{1i}^* = U_1 - U_2$. All voters with $\delta_{1i} \leq \delta_{1i}^*$ will vote for the incumbent. Thus the proportion of votes for L in province 1 is given by $\Phi_1(U_1 - U_2)$, where $\Phi_1(.)$ is the cdf of the variable $\delta_{1i}$. We make the following assumption about the distribution function.

**Assumption D1:** $\Phi_1(.)$ is symmetric around zero.

Similarly, in province 2, the indifferent voter is situated at $\delta_{2i}^* = U_2 - U_1$. The proportion of votes for the incumbent in province 2 is $\Phi_2(U_2 - U_1)$. The proportion of votes for party L in province 2 is $1 - \Phi_2(U_2 - U_1) = \Phi_2(U_1 - U_2)$.

The federal government maximizes a weighted sum of federal utility and vote share: thus it is partly benevolent and partly partisan. Let $\lambda \geq 0$ be the weight. Then the maximand of the federal government is

$$L_C = U_1 + U_2 + \lambda[\Phi_1(U_1 - U_2) + \Phi_2(U_1 - U_2)]$$

(2.4)

which is to be maximized with respect to $T_1$.\textsuperscript{12}

\textsuperscript{12}We can also consider the alternate electoral incentive of capturing half of the electorate. If we assume that in both regions, the population is normalised to 1, then the central government's objective would be

$$\max_{T_1} U_1 + U_2$$

such that $\Phi_1(.) \geq .5$

$\Phi_2(.) \geq .5$

If we assume that the central government places the same weight on winning the election

informational issues. Politicians can only manipulate the transfer levels to earn more vote. Thus the role of the voters is much like the regulatory authority in Shleifer (1986). That being said, our model do not differ from the voting behaviour discussed in Besley and Case, op. cit. pp. 32.
Finally, the provincial governments choose the effort level to maximise the vote share, which amounts to increase the utility differential. We assume the following.

**Assumption PG**: The provincial governments neglect the spillover effect.

This is a somewhat standard assumption in federalism literature. If this is true, the provincial government in region \( i \) is interested in maximising only \( U_i \).

**Comment**: This assumption can be relaxed, such that the provincial government in region \( i \) maximise

\[
U_i + \Phi_i(U_i - U_j)
\]

This implies still lower effort by region \( i \).

### 3 Characterization of Grants Under Non Commitment

In the case of non commitment, the sequence of moves is as follows. First, the province governments set \( \theta_i \). Then the federal government fixes \( T_i \). In from both regions, then the Lagrangian is

\[
L = U_1 + U_2 + \lambda[\Phi_1(.) + \Phi_2(.) - 1]
\]

For all practical purpose, this is identical with the central’s objective (2.4)
the last stage, the public good is provided and consumed.

### 3.1 Non-Political Federal transfer

Suppose we have the two stage game as described above.

Let the federal government maximize the sum of utilities. In other words, given \( \theta_i \), it maximizes

\[
\sum_i [u_i(\theta_i + T_i + \beta(\theta_j - T_i)) + g_i(Y_i - \theta_i)]
\]

with respect to the transfers. The F.O.C. yields the following, in reduced form.

\[
u'_1 = u'_2 \quad \text{(3.1)}
\]

Solving the first order conditions gives \( T_i = T_i(\theta_1, \theta_2) \). From the F.O.C.s, we have:

\[
\left( \frac{\partial T_i}{\partial \theta_i} + 1 - \beta \frac{\partial T_i}{\partial \theta_i} \right) u''_i = \left( -\frac{\partial T_i}{\partial \theta_i} + \beta \left( 1 + \frac{\partial T_i}{\partial \theta_i} \right) \right) u''_j \Rightarrow \]

\[
\frac{\partial T_i}{\partial \theta_i} = \frac{\beta u''_j - u''_i}{(1 - \beta)(u''_1 + u''_2)} \quad \text{(3.2)}
\]

This is the marginal incentive for local effort.

**Example: Linear Quadratic Utility**

\[
u_i = P_i - \frac{\eta_i P_i^2}{2}
\]

\( u'_1 = u'_2 \) implies

\[
T_1 = \frac{\eta_1 (B_1 + \beta B_2) - \eta_2 (B_2 + \beta B_1) + \theta_1 (\gamma_1 - \beta \gamma_2) + \theta_2 (\beta \gamma_1 - \gamma_2)}{(\eta_1 + \eta_2)(1 - \beta)}
\]
such that
\[
\frac{\partial T_1}{\partial \theta_1} = \frac{\beta \eta_2 - \eta_1}{(\eta_1 + \eta_2)(1 - \beta)}
\]
This is positive if \(\beta \eta_2 > \eta_1\)

3.1.1 Decomposition of Marginal Incentive:

A rise in \(\theta_i\), \textit{ceteris paribus}, increases \(p_i\). This also implies a fall in marginal utility of both provinces (since both \(P_i\) and \(P_j\) increase). The fall in the marginal utility of province \(i\) is \(u''_i\) and that in the neighboring province is \(\beta u''_j\). If the fall in province \(i\) is greater than the corresponding fall in province \(j\), then it is necessary to have a reduction in marginal incentive in province \(i\), i.e. there should be less subsidy at the margin.\(^{13}\)

Thus, we have the following:

\[
\frac{\partial T_i}{\partial \theta_i} = \frac{1}{1 - \beta} \left[ \frac{\beta u''_i}{u''_i + u''_j} - \frac{u''_1}{u''_i + u''_j} \right] \quad (3.3)
\]

In (3.3), the positive first term within the parenthesis captures the quasi-Pigouvian nature. Since the provinces confer a positive benefit on each other, they should receive a subsidy at the margin. The subsidy depends on the magnitude of the spillover.

Thus, \(\frac{\partial T_i}{\partial \theta_i} < 0\) iff the ‘equalization component’ dominates the Pigouvian component.\(^{14}\) The above discussion leads to:

\textbf{Lemma 1:} \textit{If the fall in marginal utility in any province is higher than the fall in MU of the other province (e.g. } |u''_i| > |u''_j| \textit{), and/or the spillover (\(\beta\))}

\(^{13}\)A graphical representation is provided in Appendix 1.

\(^{14}\)This decomposition is due to Koethenbuerger (2006)
is weak, then equalization component would dominate the Pigouvian component, such that for that province, the marginal incentive is negative.

In the first stage of the game, welfare-oriented provincial government in region $i$ maximizes

$$u_i(T_i + \theta_i + \beta(\theta_j + T_j)) + g_i(Y_i - \theta_i)$$

such that $T_i = T_1(\theta_1, \theta_2) = -T_2(\theta_1, \theta_2)$.

The F.O.C. yields,$^{15}$

$$u_i' \frac{\partial P_i}{\partial \theta_i} - g_i'(Y_i - \theta_i) = 0$$

or,

$$\left(1 + (1 - \beta) \frac{\partial T_i}{\partial \theta_i}\right) u_i' = g_i'$$

Comment: (3.4) provides an intuitive explanation when a province chooses to provide zero revenue. If the provinces have high ‘status quo’ level of public good, or if their initial level of income ($y_i$), are low, then $u_i'$ is low enough compared to $g_i'$ and $u_i' \frac{\partial P_i}{\partial \theta_i} - g_i' < 0$. Province $i$ chooses $\theta_i = 0$.

### 3.2 Political Transfer and Marginal Incentive

Under political transfer, the federal government maximizes both provincial welfare and vote share. The F.O.C. of (2.4) is given by:

$^{15}$Sufficiently concave $u$ and $g$ guarantee the SOC and an interior solution for $\theta_i$.
\[ u_1' - u_2' + \lambda(\Phi_1')(u_1' + u_2') + \lambda(\Phi_2')(u_1' + u_2') = 0 \]

Here we make the second simplifying assumption about the distribution.

**Assumption D2:** The distribution of \( \Phi_i \) is uniform.

Let \( \Phi_i' = \alpha_i \) (a constant), and \( \alpha = \alpha_1 + \alpha_2 \). Then, the above equation reduces to

\[(1 + \lambda\alpha)u_1' = (1 - \lambda\alpha)u_2' \]

or,

\[ u_1' = Au_2' \quad (3.5) \]

Assuming uniform distributions, \( \alpha = \Phi_1'(U_1 - U_2) + \Phi_2'(U_1 - U_2) \) is a constant. This is the marginal swing in favour of party L given an increase in difference in utilities. The S.O.C. is satisfied if \( \lambda\alpha < 1 \). Here, \( A = \frac{1 - \lambda\alpha}{1 + \lambda\alpha} \) is a fraction.

Thus, federal government appears to maximize the weighted sum of utilities, i.e. \( U_1 + AU_2 \), with \( A < 1 \). When \( A \to 1 \), we have the first best case. Note that, \( A \) goes down if either \( \alpha \) or \( \lambda \) goes up. In other words, as voters get more responsive to utility difference or the weight attached to vote share goes up, less weight is attached to region 2’s welfare.

**Comment:** In the absence of political concerns the utility effect of higher transfers to the aligned province has to be weighed against the reduction in utility in the other state. With political concerns a higher transfer to
the aligned province has additional positive effects (and only positive effects from the perspective of the federal government). The reduction in utility in province 2 increases the vote share in both states and the increase in utility in province 1 further increase the vote shares of the federal party. Put differently, the central government has a political incentive to fiscally exploit province 2 which amounts to putting less weight on province 2’s utility in the maximization problem.\footnote{I am grateful to one of the referees to point this out.}

Solving (3.5), we get \( T_i = T_i(\theta_1, \theta_2; A) \). Therefore \( T_i \) depends on \( \theta_i \) (the incentive effect) as well as \( A \) (the scale effect). The following lemma shows that, \textit{ceteris paribus}, the scale effect is always negative: if either \( \lambda \) or \( \alpha \) increases, such that \( A \) goes down, then the level of transfer to region 1 goes up.

\textbf{Lemma 2:} \textit{Ceteris paribus}, decreasing political weight to the non-favorite region, the level of transfer to favorite region goes up.

\[ (1 - \beta) \left( \frac{\partial T_1}{\partial A} \right)_P = \frac{u'_2}{u'_1 + Au'_2} < 0 \]

The subscript “\( P \)” refers to "political" transfer. With increasing political weight (\( \alpha \)), province 1 enjoys an increase in subsidy. The logic behind this can be seen from (3.5). As \( \alpha \) goes up, \textit{ceteris paribus}, the LHS falls and RHS goes up. To restore the balance, there should be more transfer to province 1 and reduced transfer (or more tax) to province 2.
3.2.1 Effect on Marginal Incentive

Differentiating $T_1$ with respect to $\theta_1$, we get

$$\left( \frac{\partial T_1}{\partial \theta_1} \right)_P = \frac{\beta A \omega_2'' - \omega_1''}{(1 - \beta) (\omega_2'' A + \omega_1'')} \tag{3.6}$$

This is the marginal incentive of provincial efforts when the transfers are dictated by political considerations.

$$\left( 1 - \beta \right) \left( \frac{\partial T_1}{\partial \theta_1} \right)_P = \left[ \frac{\beta A \omega_2' - \omega_1'}{\omega_2'' A + \omega_1'} - \frac{\omega_1''}{\omega_2'' A + \omega_1''} \right]$$

**Example:** With linear quadratic utility, $(1 - \beta) \left( \frac{\partial T_1}{\partial \theta_1} \right)_P = \frac{\beta A \eta_2 - \eta_1}{\eta_2 A + \eta_1}$

Similarly we have\(^\text{17}\)

$$\frac{\partial T_2}{\partial \omega_2} = \frac{\beta A \omega_2'' - \omega_1''}{(1 - \beta) (\omega_2'' A + \omega_1'')} \quad \text{and} \quad \frac{\partial P_1}{\partial A} = (1 - \beta) \left( \frac{\partial T_1}{\partial A} \right) = \frac{\omega_2'}{\omega_2'' A + \omega_1''} < 0$$

**Corollary:**

(a) $\frac{\partial P_1}{\partial \omega_1} = 1 + (1 - \beta) \left( \frac{\partial T_1}{\partial \omega_1} \right) = 1 + \frac{\beta A \omega_2'' - \omega_1''}{\omega_2'' A + \omega_1''} = (1 + \beta) \frac{A \omega_2''}{\omega_2'' A + \omega_1''} \geq 0$

(b) $\frac{\partial P_1}{\partial \omega_2} = \beta + (1 - \beta) \left( \frac{\partial T_1}{\partial \omega_2} \right) = \beta - \frac{\beta A \omega_2'' - \omega_1''}{\omega_2'' A + \omega_1''} = (1 + \beta) \frac{A \omega_2''}{\omega_2'' A + \omega_1''} = \frac{\partial P_1}{\partial \omega_1}$

(c) $\frac{\partial P_2}{\partial \omega_2} = \frac{\partial P_2}{\partial \omega_1} = 1 + (1 - \beta) \frac{\partial T_2}{\partial \omega_2} = (1 + \beta) \frac{A \omega_2''}{\omega_2'' A + \omega_1''} \geq 0$

(d) $\frac{\partial P_1}{\partial A} = -\frac{\partial P_2}{\partial A}$

**Proof.** These observations follow from $P_i = \theta_i + \beta \theta_j + (1 - \beta) T_i$ and $T_i = -T_j$. ■

**Comment:** Thus,

(a) The presence of political factor makes the spillover/ Pigouvian term weak (as observed in the numerator of the first term, $A\beta$ instead of $\beta$).

(b) On the other hand, this boosts the Pigouvian element as well as equalization component. This effect is manifested in the denominators of the

\(^{17}\text{We drop the subscript } P \text{ for notational clarity.}\)
fractions. Since it affects both the components equally, the net effects cancel out. Whether or not the presence of party politics increases or reduces the marginal incentive for effort is, therefore, ambiguous and depends on the structure of the utility function. This is discussed in the next section.

3.2.2 The Sign Of Cross Derivatives:

We express $T_i = T_i(\theta_1, \theta_2, A)$. Thus, the marginal incentive to any region, say region 1 is a function of these variables.

Local Effort and Marginal Incentive: We have

$$\frac{\partial P_1}{\partial \theta_1} = 1 + (1 - \beta) \frac{\partial T_1}{\partial \theta_1} = \frac{\nu_1'}{\nu_1^2 A + \nu_1'} A (1 + \beta)$$

$$(1 - \beta) \frac{\partial^2 T_1}{\partial \theta_1^2} = A (1 + \beta) \frac{u_1'' u_2' (\phi_1 \frac{\partial P_1}{\partial \theta_1} - \phi_2 \frac{\partial P_2}{\partial \theta_1})}{(u_2'' A + u_1'')^2}$$

Here, $\phi_i = -\frac{u_1''}{u_1'}$ = Coefficient of absolute prudence in province $i \geq 0$

This is zero for linear quadratic function.\(^\text{18}\)

Similarly,

$$\frac{\partial^2 P_2}{\partial \theta_2^2} = (1 - \beta) \frac{\partial^2 T_2}{\partial \theta_2^2} = -(1 - \beta) \frac{\partial^2 T_1}{\partial \theta_2^2} = -\frac{\partial^2 P_1}{\partial \theta_1^2}$$

**Corollary:** If $\frac{\partial^2 P_1}{\partial \theta_1^2}$ is positive, negative, or zero then $\frac{\partial^2 P_2}{\partial \theta_2^2}$ is negative, positive, or zero.

\(^{18}\)This is zero for logarithmic utility function $u_i = \gamma_i P_i$ as well as CARA utility function $u_i = -\sigma_i \exp(-\sigma_i P_i)$. 

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Neighboring Effort and Marginal Incentive:

\[
(1 - \beta) \frac{\partial^2 T_1}{\partial \theta_1 \partial \theta_2} = \frac{A(1 + \beta)u''_1u''_2 (\phi_1 \frac{\partial P_2}{\partial \theta_2} - \phi_2 \frac{\partial P_1}{\partial \theta_2})}{(u''_1 + Au''_2)^2}
\]

Thus, \( (1 - \beta) \frac{\partial^2 T_1}{\partial \theta_1 \partial \theta_2} = (1 - \beta) \frac{\partial P_2}{\partial \theta_2} \). The same conclusion as the above follows.

Political Variable and Marginal Incentive:

\[
\frac{\partial}{\partial A} \left( \frac{\partial T_1}{\partial \theta_1} \right) = \frac{(u''_2 A + u''_1) \frac{\partial (\beta u''_2 - u''_1)}{\partial A} - (\beta u''_2 - u''_1) \frac{\partial \phi_1 + \phi_2}{\partial A}}{(u''_2 A + u''_1)^2}
\]

The sign depends on the sign of the numerator of the right hand side expression.

Expanding the numerator,

\[
Au''_1u''_2(1 - \beta^2) \frac{\partial T_1}{\partial A} (\phi_1 + \phi_2) + u''_1u''_2(1 + \beta)
\]

For this to be positive

\[
Au''_1u''_2(1 - \beta^2) \frac{\partial T_1}{\partial A} (\phi_1 + \phi_2) + u''_1u''_2(1 + \beta) > 0
\]

or

\[
A(1 - \beta) \frac{\partial T_1}{\partial A} (\phi_1 + \phi_2) + 1 > 0
\]

or

\[
\frac{Au''_2}{u''_1 + Au''_2} (\phi_1 + \phi_2) + 1 > 0 \quad (3.7)
\]

Similarly, we have

\[
\frac{\partial^2 P_2}{\partial \theta _2 \partial A} = - (1 - \beta) \frac{\partial}{\partial A} \left( \frac{\partial T_1}{\partial \theta_2} \right) = \frac{(u''_1 + Au''_2) \frac{\partial (Au''_2 - u''_1)}{\partial A} - (Au''_2 - u''_1) \frac{\partial (u''_2 + Au''_2)}{\partial A}}{(u''_1 + Au''_2)^2}
\]

The numerator equals

\[
u''_1u''_2(1 + \beta) + Au''_1 \frac{\partial u''_2}{\partial A} (1 + \beta) - Au''_2 \frac{\partial u''_1}{\partial A} (1 + \beta), \text{ which is same as above.}
\]

The observation can be put in the following lemma.

**Lemma 3:** If either (a) \( A \) is too low or (b) the coefficients of absolute prudence is high, then the marginal incentive to region 1 increases with lower
Let, as in the linear quadratic case, $\phi_i = 0$. Then $\frac{\partial^2 P_i}{\partial \theta_1 \partial A} = \frac{1-\beta}{(u_2^0 A + u_1^0)^2} > 0$, in other words, the marginal incentive to region 1 decreases with lower political weight to region 2.\footnote{For CARA utility function, $\phi_1 = \sigma_i$. It can be shown that $\frac{\partial^2 P_i}{\partial \theta_1 \partial A} = 0$ for all $\sigma_i$. For logarithmic utility, $\frac{\partial^2 P_i}{\partial \theta_1 \partial A} < 0$ that is, the marginal incentive to region 1 increases as $A$ decreases.}

4 Provincial Revenue, Public Good and Welfare

In this section, we investigate the effects of political variable on provincial revenue raising efforts, provincial and federal production of public good as well as provincial and federal welfare.

4.1 Provincial Revenue Raising Efforts:

As before, welfare maximizing provincial governments maximize $u_i (.) + g_i(.)$ by choosing $\theta_i$ and taking $T_i(\theta_i, \theta_j; A)$ as given.

The F.O.C. of the provincial government in region 1 is

$$\left(1 + (1 - \beta) \left( \frac{\partial T_1}{\partial \theta_1} \right)_P \right) u_1' (P_1) = g_1'(y_1 - \theta_1)$$

And, that for region 2

$$\left(1 - (1 - \beta) \left( \frac{\partial T_2}{\partial \theta_2} \right)_P \right) u_2' (P_2) = g_2'(y_2 - \theta_2)$$

The provincial level equations are
\[
\frac{\partial P_1}{\partial \theta_1} u'_1(P_1) = g'_1(y_1 - \theta_1) \\
\frac{\partial P_2}{\partial \theta_2} u'_2(P_2) = g'_2(y_2 - \theta_2)
\] (4.1)

One can express the provincial effort levels as (using the implicit function theorem) \(\theta_i = R_i(\theta_j; A)\).

The following proposition summarizes the sufficient condition for provincial efforts to be strategic substitutes.

**Proposition 1** If the cross derivatives of the transfer with respect to efforts are zero, the effort levels are strategic substitutes. In other words, \(\frac{d\theta_i}{d\theta_j}\big|_{\theta_i=R_i(\theta_j;A)} < 0\). In general,

\[
\frac{d\theta_i}{d\theta_j}\big|_{\theta_i=R_i(\theta_j;A)} = \frac{A_i}{SOC_i}
\]

Where,

\[
A_i = -u'_i \frac{\partial^2 P_i}{\partial \theta_1 \partial \theta_2} u''_i \left( \frac{\partial P_i}{\partial \theta_i} \right)^2
\]

**Proof.** See appendix. ■

**Comment:** For stability of the equilibrium, it must be true that

\[-1 < \left( \frac{A_1}{SOC_1} \right) \left( \frac{A_2}{SOC_2} \right) < 1\] (4.2)

Finally, the effect of the political transfer on provincial revenue raising efforts is summarized in the following lemma

**Lemma 4:** In general,
\[
\frac{d\theta_i}{dA} = \frac{\Omega_i (SOC_i) + \Omega_j A_i}{|D|}
\]

Where

\[
\Omega_i = -\left( \frac{\partial P_i}{\partial A} \right) \left( \frac{\partial P_i}{\partial \theta_i} \right) u'' \cdot \frac{\partial^2 P_i}{\partial \theta_i \partial A} u'_i
\]

and \(|D| = (SOC_1)(SOC_2) - A_1 A_2 > 0\) by stability conditions (4.2)

**Proof.** See appendix: ■

The proposition immediately follows.

**Proposition 2:** Let the utility functions be such that (a) provincial revenue raising efforts are strategic substitutes and (b) marginal incentive in favorite province falls with decreasing \(A\), while that in the non-favorite province increase. Then \(\frac{d\theta_1}{dA} > 0\) and \(\frac{d\theta_2}{dA} < 0\).

**Proof.** The first part of the proposition tells us that \(A_i > 0\). The second sufficient condition implies \(\frac{\partial^2 P_1}{\partial \theta_1 \partial A} > 0\) and \(\frac{\partial^2 P_2}{\partial \theta_2 \partial A} < 0\). Thus, \(\Omega_1 < 0\) and \(\Omega_2 > 0\) ⇒ \(\Omega_1 (SOC_2) + \Omega_2 A_1 > 0\) and \(\Omega_2 (SOC_1) + \Omega_1 A_2 < 0\).

Together with lemma 4, these observations simply mean \(\frac{d\theta_1}{dA} > 0\) and \(\frac{d\theta_2}{dA} < 0\). ■

**Example:**

a) For linear quadratic utility, \(A_1 > 0\), \(A_2 > 0\), \(\Omega_1 < 0\), \(\Omega_2 > 0\). Thus \(\Omega_1 (SOC_2) + \Omega_2 A_1 > 0\) and \(\Omega_2 (SOC_1) + \Omega_1 A_2 < 0\). Thus, decreasing political weight to region 2 decreases the revenue in region 1 and increases that in region 2.

b) For CARA utility functions,\(^{20}\) \(A_1 > 0\), \(A_2 > 0\), \(\Omega_1 < 0\), \(\Omega_2 > 0\). Thus \(\frac{d\theta_1}{dA} > 0\) and \(\frac{d\theta_2}{dA} < 0\). Thus the same conclusion follows.

\(^{20}u_i(P_i) = -\sigma_i e^{-\sigma_i P_i}\)
c) For logarithmic utility, \( A_1 > 0 \), \( A_2 > 0 \) while \( \Omega_1 \) and \( \Omega_2 \) are ambiguous. It turns out that, \( \Omega_i (SOC_j) + \Omega_j A_i = 0 \), such that \( \frac{d\Omega_i}{dA} = 0 \)

i.e. political weight does not have any effect on revenue raising effort of the local governments.

### 4.2 Production of Public Good:

Since \( P_1 = \theta_1 + \beta \theta_2 + (1 - \beta) T_1 \), taking total differential with respect to \( A \), we get the effect of political variable on the consumption of public good in the politically favorite region

\[
\frac{dP_1}{dA} = \frac{d\theta_1}{dA} + \beta \frac{d\theta_2}{dA} + (1 - \beta) \left( \frac{dT_1}{d\theta_1} \frac{d\theta_1}{dA} + \frac{dT_2}{d\theta_2} \frac{d\theta_2}{dA} + \frac{dT_1}{dA} \right)
\]

\[
= \left( 1 + (1 - \beta) \frac{dT_1}{d\theta_1} \right) \frac{d\theta_1}{dA} + \left( \beta + (1 - \beta) \frac{dT_2}{d\theta_2} \right) \frac{d\theta_2}{dA} + (1 - \beta) \frac{dT_1}{dA}
\]

\[
= \frac{dP_1}{d\theta_1} \left( \sum \frac{d\theta_1}{dA} \right) + \frac{dP_2}{d\theta_2} \left( \sum \frac{d\theta_2}{dA} \right)
\]

Note that, \( \frac{dP_1}{dA} < 0 \). We call this the partisan effect of political variable: with increasing \( A \), region 1 will have higher public good.

The scale effect is complemented by the local effects, that is, \( \sum \frac{d\theta_1}{dA} \). If \( \sum \frac{d\theta_1}{dA} \leq 0 \), \( \frac{dP_1}{dA} < 0 \). In other words, if the total effort increases as \( A \) falls, the consumption of public good in region 1 increases.

The total public good production within the economy is given by \( P_1 + P_2 = (1 + \beta)(\theta_1 + \theta_2) \). Thus, \( \sum \frac{dP}{dA} = (1 + \beta) \left( \sum \frac{d\theta_1}{dA} \right) \). Thus \( \text{sign} \left( \sum \frac{dP}{dA} \right) = \text{sign} \left( \sum \frac{d\theta_1}{dA} \right) \). If the total revenue falls with decreasing \( A \), the total public good production also gets reduced.

**Proposition 3:** Suppose the conditions on the utility function as listed in proposition 2 hold. The total amount of public good produced within the federation increases (decreases) with \( A \) if the marginal cost of revenues in

\[ u_i (P_i) = \gamma_i \ln P_i \]
the second province is high (low) enough.

Proof. See appendix. The proof immediately follows from the fact that
\[ \sum P_i = (1 + \beta) \sum \theta_i \Rightarrow \sum \frac{dP_i}{dA} = (1 + \beta) \left( \sum \frac{d\theta_i}{dA} \right). \]

Comment: If \( \sum \frac{dP_i}{dA} > 0 \), then with decreasing \( A \), the total production of public good falls. Under politically motivated transfer scheme, revenue generation in the second region increases, while that in the first region falls. If the cost in the second region is very high, then the shortfall in revenue from the first province is not compensated by increased revenue from the second region.

The above proposition can be stated more sharply if we assume a linear quadratic formulation with identical utility functions, e.g.

\[ u_i'' = -\eta \quad \text{(4.3)} \]
\[ g_i'' = -\delta_i \]

The alternate form can be stated as

**Lemma 5** For linear quadratic utility functions such as above, \( \sum \frac{dP_i}{dA} > 0 \) iff \( A\delta_2 > \delta_1 \). For symmetric cost functions (\( \delta_1 = \delta_2 \)), \( \sum \frac{dP_i}{dA} < 0 \) and the total provision of public good rises with decreasing \( A \).

Proof. See appendix. ■

Comment: Even if \( \delta_2 \) is high, a low \( A \) ensures that \( \sum \frac{dP_i}{dA} < 0 \), i.e. the total public good rises with decreasing \( A \).

The following proposition lists the necessary conditions for the public good consumption in the favourite province to fall as \( A \) decreases. To get sharper results, we continue with the linear quadratic formulation with iden-
tical utility functions.

**Lemma 6:** Necessary conditions for \( \frac{dP_1}{dA} > 0 \) are given by \( A\delta_2 > \delta_1 \) and a sufficiently large \( \eta \).

**Proof.** See appendix. ■

**Comment:** As noted earlier, the centre reduces revenue in region 1 and increases that in region 2. If \( \delta_2 \) is large, then the cost in the second province is large (which prohibits the centre, as a welfare maximiser, from exploiting the second province too much). If \( \eta \) is large, then increase in \( P_2 \) will not result in large increase in \( u_2 \). Since creating a difference in utility pays to the central authority the net result is a reduction in \( \theta_1 \) and an increase in \( \theta_2 \) such that \( P_1 \) falls and \( P_2 \) increases.

### 4.3 Welfare Analysis:

The welfare of the favorite province is \( U_1 = u_1 (P_1) + g_1 (y - \theta_1) \). Thus, \( \frac{dU_1}{dA} = u_1' \frac{dP_1}{dA} - g_1' \frac{d\theta_1}{dA} \). For \( \frac{dU_1}{dA} < 0 \) (that is, welfare to region 1 increases as \( A \) falls) it must be the case that \( \frac{d\theta_1}{dA} > 0 \) (that is, effort in the favourite region falls as \( A \) decreases) and \( \frac{dP_1}{dA} < 0 \) i.e. \( P_1 \) increases as \( A \) falls. If the utility function exhibits property such that (a) provincial revenue raising efforts are strategic substitutes and (b) the marginal incentive to region 1 decreases as \( A \) falls, then \( \frac{d\theta_1}{dA} > 0 \). The formal statement is summarised in the following lemma.

**Proposition 4:** Suppose the utility functions obey the sufficient conditions as listed in proposition 2. Then, \( \frac{dU_1}{dA} < 0 \).

**Proof.** We know that \( u_1' \frac{dP_1}{d\theta_1} = g_1' \) (from the provincial first order conditions). Thus
\[
\frac{dU_1}{dA} = u'_1 \frac{dP_1}{dA} - u'_1 \frac{\partial P_1}{\partial \theta_1} \frac{d\theta_1}{dA} \\
= u'_1 \left( \frac{dP_1}{dA} - \frac{\partial P_1}{\partial \theta_1} \frac{d\theta_1}{dA} \right)
\]

But \( \frac{dP_1}{dA} = \frac{\partial P_1}{\partial \theta_1} \left( \sum \frac{d\theta_i}{dA} \right) + \frac{\partial P_1}{\partial A} \), thus
\[
\frac{dU_1}{dA} = u'_1 \left( \sum \frac{d\theta_i}{dA} \right) + u'_1 \frac{\partial P_1}{\partial A} + u'_1 \frac{\partial P_1 \partial \theta_1}{\partial A} \]

Since \( \frac{\partial P_1}{\partial \theta_1} > 0 \) and \( \frac{\partial P_1}{\partial A} < 0 \). We know, if the utility exhibits the sufficient conditions as mentioned in proposition 2, then
\( \frac{dU_1}{dA} < 0 \).

**Comment:** The above proposition holds true even if the public good falls in region 1.

The total welfare within the economy is

\[
W = U_1 + U_2
\]

Thus
\[
\frac{dW}{dA} = \frac{dU_1}{dA} + \frac{dU_2}{dA}
\]
\[
= u'_1 \left( \sum \frac{d\theta_i}{dA} \right) + u'_1 \frac{\partial P_1}{\partial A} + u'_1 \frac{\partial P_1 \partial \theta_1}{\partial A} + \frac{dP_2}{dA} - \frac{\partial P_2}{\partial \theta_2} \frac{d\theta_2}{dA}
\]
\[
= u'_2 \left[ \frac{\partial P_1}{\partial A} \left( A - 1 \right) + \frac{\partial P_2}{\partial \theta_2} \frac{d\theta_2}{dA} + A \frac{\partial P_1}{\partial \theta_1} \frac{d\theta_1}{dA} \right]
\]

The first term within the parenthesis is positive as \( A < 1 \). Under the restrictions on utility functions as discussed in proposition 2, \( \frac{\partial \theta_1}{dA} > 0 \) and \( \frac{\partial \theta_2}{dA} < 0 \). The term within the parenthesis is thus ambiguous. However, with
using linear quadratic specifications as listed under (4.3), we obtain a useful result which is summarized below.

**Lemma 7**: If the marginal cost in the first region is higher than that of second, then there exists a range for $A$ where the federal welfare increases as $A$ goes down.

**Proof.** See appendix.

This suggests that if the cost in the favourite province is high enough, then the effect of reducing $A$ on federal utility is ambiguous. More specifically, federal utility increases with decreasing $A$ in the vicinity of $A = 1$. However, when $\lim A \to 0$, welfare falls as $A$ decreases. At least for such families of utility functions, politically motivated transfers can be welfare enhancing, for a range of $A$.

![Figure 1: Political Variable and Welfare: $\delta_2 > \delta_1$](image)

5 Conclusion

In this paper, we have examined how party politics alters the provision and distribution of a public good within a federation. The provinces produce a public good (having interprovincial spillover effects) with federal funds and
provincial revenue. The revenue raising efforts (which are taxes on voters) are costly. Voters’ utility depends on public good and the cost of effort. Since voters view the provincial differential in utility as a measure of competence of their governments, vote shares for each party is a function of the difference between provincial utilities. The federal government maximizes a weighted sum of federal utility (which is, the sum of provincial welfare) and the vote share (which is a function of difference between provincial welfare).

We have demonstrated the following facts. The presence of political factor has a partisan effect on public good provision such that, ceteris paribus, the favourite province receives more transfer and enjoys more public good. On the other hand, such a transfer may alter incentives for raising provincial revenue. The incentive effect complements the partisan effect of transfers. For a class of utility functions, increased political element in transfers reduce the effort in region 1 and increases that in region 2. It may be possible that the public good consumption in the politically favourite province falls as transfers become more and more politically motivated, especially if the utility functions are linear quadratic in nature. The federal production of public good, which is the sum of provincial revenues, may increase or decrease with the political element. The welfare in the favourite province always increases. For linear quadratic utility functions, the federal welfare (sum of the provincial welfare) may actually increase with politically motivated transfer.

This analysis can be extended in several directions. First, different protocols of interaction between the provinces (such as cooperative or bargaining solution) may lead to different results. Second, one can introduce dynamic provision of a public good, where the provincial and federal governments en-
gage in a differential game. A testable hypothesis also emerges: the presence of politics increases the level of grants to a favourite province, but there may not be any significant correlation with the level of public good consumption across different provinces.
Appendix

Proof of Propositions and Lemmas

Proof of Proposition 1

The equation for the first province is

\[ \frac{\partial P_1}{\partial \theta_1} u'_1(P_1) = g'_1(y_1 - \theta_1) \]

Differentiating w.r.t. \( \theta_1 \)

\[ \frac{d\theta_1}{d\theta_2} \left[ u'_1 \frac{\partial^2 P_1}{\partial \theta_1^2} + u''_1 \left( \frac{\partial P_1}{\partial \theta_1} \right)^2 + g''_1 \right] = -u'_1 \frac{\partial^2 P_1}{\partial \theta_1 \partial \theta_2} - u''_1 \frac{\partial P_1}{\partial \theta_1} \frac{\partial P_1}{\partial \theta_2} \]

Here,

\[ \frac{\partial P_1}{\partial \theta_1} = 1 + (1 - \beta) \frac{\partial T_1}{\partial \theta_1} = 1 + \frac{A \beta u''_2 - u''_1}{A u''_2 + u''_1} = \frac{A u''_2(1 + \beta)}{A u''_2 + u''_1} > 0 \]

and

\[ \frac{\partial P_1}{\partial \theta_2} = \beta + (1 - \beta) \frac{\partial T_1}{\partial \theta_2} = \beta + \frac{A u''_2 - \beta u''_1}{A u''_2 + u''_1} = \frac{A u''_2(1 + \beta)}{A u''_2 + u''_1} = \frac{\partial P_1}{\partial \theta_1} > 0 \]

Thus

\[ \frac{d\theta_1}{d\theta_2} \left[ u'_1 \frac{\partial^2 P_1}{\partial \theta_1^2} + u''_1 \left( \frac{\partial P_1}{\partial \theta_1} \right)^2 + g''_1 \right] = -u'_1 \frac{\partial^2 P_1}{\partial \theta_1 \partial \theta_2} - u''_1 \left( \frac{\partial P_1}{\partial \theta_1} \right)^2 \]

or

\[ \frac{d\theta_1}{d\theta_2} \bigg|_{\theta_1 = R_1(\theta_2; A)} = \frac{-u'_1 \frac{\partial^2 P_1}{\partial \theta_1 \partial \theta_2} - u''_1 \left( \frac{\partial P_1}{\partial \theta_1} \right)^2}{u'_1 \frac{\partial^2 P_1}{\partial \theta_1^2} + u''_1 \left( \frac{\partial P_1}{\partial \theta_1} \right)^2 + g''_1} = \frac{A_1}{SOC_1} \]
This can be positive, negative or zero. If \( \frac{\partial^2 P_1}{\partial \theta_1 \partial \theta_2} = (1 - \beta) \frac{\partial^2 T_1}{\partial \theta_1 \partial \theta_2} \leq 0 \), then \( A_1 \) is positive. In other words, for LQ, logarithmic and CARA utility functions, \( \frac{d \theta_1}{d \theta_2} \bigg|_{\theta_1 = R_1(\theta_2;A)} < 0 \)

Similarly, differentiating
\[
\frac{\partial P_2}{\partial \theta_2} u'_2(P_2) = g'_2(y_2 - \theta_2)
\]

Differentiating

Thus

\[
\frac{d \theta_2}{d \theta_1} \bigg|_{\theta_2 = R_2(\theta_1;A)} = \frac{-\frac{\partial^2 P_2}{\partial \theta_1 \partial \theta_2} u'_2}{u'_2 \frac{\partial^2 P_2}{\partial \theta_2^2} + \left( \frac{\partial P_2}{\partial \theta_2} \right)^2 u''_2 + g''_2} = \frac{A_2}{SOC_2}
\]

Now \( \frac{\partial^2 P_2}{\partial \theta_1 \partial \theta_2} = -\frac{\partial^2 P_1}{\partial \theta_1 \partial \theta_2} \). Thus, if \( \frac{d \theta_1}{d \theta_2} \bigg|_{\theta_1 = R_1(\theta_2;A)} > 0 \rightarrow \frac{\partial^2 P_1}{\partial \theta_1 \partial \theta_2} \geq 0 \),
then \( \frac{\partial^2 P_2}{\partial \theta_1 \partial \theta_2} \leq 0 \rightarrow A_2 > 0 \Rightarrow \frac{d \theta_2}{d \theta_1} \bigg|_{\theta_2 = R_2(\theta_1;A)} < 0 \). However,

if \( \frac{\partial^2 P_1}{\partial \theta_1 \partial \theta_2} < 0 \), then signs are ambiguous.

Again, if \( \frac{\partial^2 P_2}{\partial \theta_1 \partial \theta_2} = 0 \rightarrow A_2 = -\left( \frac{\partial P_2}{\partial \theta_2} \right)^2 u''_2 > 0 \), then \( \frac{d \theta_2}{d \theta_1} \bigg|_{\theta_2 = R_2(\theta_1;A)} < 0 \)

0

**Proof of Lemma 4**

Differentiating the reaction function of the first province with respect to \( A \),

\[
u'_1(P_1) \left[ \frac{\partial^2 P_1}{\partial \theta_1^2} \frac{d \theta_1}{d A} + \frac{\partial^2 P_1}{\partial \theta_1 \partial \theta_2} \frac{d \theta_2}{d A} + \frac{\partial^2 P_1}{\partial \theta_2^2} \right] + \frac{\partial P_1}{\partial \theta_1} u''_1(P_1) \left[ \frac{\partial P_1}{\partial \theta_1} \frac{d \theta_1}{d A} + \frac{\partial P_1}{\partial \theta_2} \frac{d \theta_2}{d A} + \frac{\partial P_1}{\partial A} \right] = \]

\[-g'_P \frac{d \theta_1}{d A} u'_1(P_1) \left[ \frac{\partial^2 P_1}{\partial \theta_1^2} + \left( \frac{\partial P_1}{\partial \theta_1} \right)^2 u''_1 + g''_P \right] + \frac{d \theta_2}{d A} \left[ u'_1 \frac{\partial P_1}{\partial \theta_1 \partial \theta_2} + \left( \frac{\partial P_1}{\partial \theta_1} \right)^2 \right] = -\left( \frac{\partial P_1}{\partial A} \right) \left( \frac{\partial P_1}{\partial \theta_1} \right) u''_1 - \]

or

30
\[
\frac{\partial}{\partial A} [SOC_1] + \frac{\partial}{\partial A} [-A_1] = \Omega_1
\]

Where \( \Omega_1 = -\left( \frac{\partial P_1}{\partial A} \right) \left( \frac{\partial P_1}{\partial x_1} \right) u''_1 - \frac{\partial^2 P_2}{\partial \sigma_1 \partial A} u'_1 \)

\[
\frac{\partial^2 P_2}{\partial \sigma_1 \partial A} = (1 + \beta) u''_1 u'_2 \left( 1 + A \frac{\partial P_2}{\partial \sigma_2} (\phi_1 + \phi_2) \right) \frac{u''_2}{(Au''_2 + u''_1)^2}
\]

- For linear quadratic utility, \( \frac{\partial^2 P_2}{\partial \sigma_1 \partial A} > 0 \), so \( \Omega_1 < 0 \).

- For CARA, \( \frac{\partial^2 P_2}{\partial \sigma_1 \partial A} = 0 \), thus \( \Omega_1 = -\left( \frac{\partial P_1}{\partial A} \right) \left( \frac{\partial P_1}{\partial x_1} \right) u''_1 < 0 \)

- For logarithmic utility, \( \frac{\partial^2 P_2}{\partial \sigma_1 \partial A} < 0 \), and thus \( \Omega_1 \) is ambiguous. If the value is high enough, then \( \Omega_1 > 0 \)

Differentiating the second equation with respect to \( A \),

\[
u'_2 \left( P_2 \right) \left[ \frac{\partial^2 P_2}{\partial \sigma_2 \partial A} + \frac{\partial^2 P_2}{\partial \sigma_1 \partial \sigma_2} \frac{\partial A}{\partial A} + \frac{\partial^2 P_2}{\partial \sigma_2} \right] + \frac{\partial P_2}{\partial \sigma_2} \left( \frac{\partial P_2}{\partial \sigma_2} \right)^2 + \frac{\partial P_2}{\partial \sigma_2} \frac{\partial A}{\partial A} = -\frac{\partial P_2}{\partial \sigma_2} \frac{\partial A}{\partial A} \frac{u''_2}{(Au''_2 + u''_1)^2}
\]

Using \( \frac{\partial P_2}{\partial \sigma_2} = \frac{\partial P_2}{\partial x_1} \),

\[
\frac{d}{dA} \left[ \nu'_2 \left( P_2 \right) \frac{\partial^2 P_2}{\partial \sigma_2} + \frac{\partial^2 P_2}{\partial \sigma_1 \partial \sigma_2} \frac{\partial A}{\partial A} + \frac{\partial^2 P_2}{\partial \sigma_2} \right] \left[ \nu'_2 \left( P_2 \right) \frac{\partial^2 P_2}{\partial \sigma_2} \right] + \left( \frac{\partial P_2}{\partial \sigma_2} \right)^2 + \frac{\partial P_2}{\partial \sigma_2} \frac{\partial A}{\partial A} = -\frac{\partial P_2}{\partial \sigma_2} \frac{\partial A}{\partial A} \frac{u''_2}{(Au''_2 + u''_1)^2}
\]

or

\[
\frac{d}{dA} [-A_2] + \frac{d}{dA} [SOC_2] = \Omega_2
\]

where, \( \Omega_2 = -\frac{\partial P_2}{\partial A} \frac{\partial P_2}{\partial x_2} u''_2 - \frac{\partial^2 P_2}{\partial \sigma_2 \partial A} u'_2
\]

\[
\frac{\partial^2 P_2}{\partial \sigma_2 \partial A} = -(1 + \beta) u''_1 u'_2 \left( 1 + A \frac{\partial P_2}{\partial \sigma_2} (\phi_1 + \phi_2) \right) \frac{u''_2}{(Au''_2 + u''_1)^2}
\]

- For linear quadratic utility, \( \frac{\partial^2 P_2}{\partial \sigma_2 \partial A} > 0 \), so \( \frac{\partial^2 P_2}{\partial \sigma_2 \partial A} < 0 \) \( \Rightarrow \Omega_2 > 0 \)

- For CARA, \( \frac{\partial^2 P_2}{\partial \sigma_2 \partial A} = 0 \), so \( \Omega_2 > 0 \)

- For logarithmic utility, \( \frac{\partial^2 P_2}{\partial \sigma_2 \partial A} < 0 \), and so \( \Omega_2 \) is ambiguous. If \( \frac{\partial^2 P_2}{\partial \sigma_2 \partial A} \) is high enough, then \( \Omega_2 \) can be negative as well.
The equations can be written as

\[
\begin{bmatrix}
SOC_1 & -A_1 \\
-A_2 & SOC_2
\end{bmatrix}
\begin{bmatrix}
\frac{d\theta_1}{dA} \\
\frac{d\theta_2}{dA}
\end{bmatrix}
= \begin{bmatrix}
\Omega_1 \\
\Omega_2
\end{bmatrix}
\]  

(A.1)

Solving, we get

\[
\frac{d\theta_1}{dA} = \frac{\Omega_1 (SOC_2) + \Omega_2 A_1}{|D|}
\]

\[
\frac{d\theta_2}{dA} = \frac{\Omega_2 (SOC_1) + \Omega_1 A_2}{|D|}
\]

**Proof of Proposition 3**

\[
\sum \frac{d\theta_i}{dA} = \Omega_1 (SOC_2) \left( 1 + \frac{A_2}{SOC_2} \right) + \Omega_2 (SOC_1) \left( 1 + \frac{A_1}{SOC_1} \right)
\]

For the utility functions for which the efforts are strategic substitutes.

\[
A_i = -u_i'' \left( \frac{\partial P_i}{\partial \theta_i} \right)^2
\]

\[
SOC_i = u_i'' \left( \frac{\partial P_i}{\partial \theta_i} \right)^2 + g_i''
\]

\[
1 + \frac{A_i}{SOC_i} = \frac{u_i'' \left( \frac{\partial P_i}{\partial \theta_i} \right)^2 + g_i'' - u_i'' \left( \frac{\partial P_i}{\partial \theta_i} \right)^2}{u_i'' \left( \frac{\partial P_i}{\partial \theta_i} \right)^2 + g_i''} = \frac{g_i''}{u_i'' \left( \frac{\partial P_i}{\partial \theta_i} \right)^2 + g_i''}
\]

So

\[
\Omega_j (SOC_i) \left( 1 + \frac{A_j}{SOC_i} \right) = \Omega_j g_i''
\]

\[
\sum \frac{d\theta_i}{dA} = \Omega_1 g_2'' + \Omega_2 g_1'' = \frac{|D|}{|D|} = 1
\]

We can write

\[
\Omega_1 = -\frac{u_i'' u_2'' (1 + \beta)}{(Au_2'' + u_1'')^2} [Au_2' + x] < 0
\]

where, \( x = 1 + \frac{\partial P_1}{\partial A} \Phi_1 + \Phi_2 \) > 0 when the marginal incentive in the favourite region falls with decreasing \( A \). Similarly,

\[
\Omega_2 = \frac{u_i'' u_1'' (1 + \beta)}{(Au_2'' + u_1'')^2} [u_1' + x] > 0
\]

For linear quadratic case, \( x = 1 \)

Dividing \( \Omega_1 \) by \( \Omega_2 \), we can express the relationship between \( \Omega_1 \) and \( \Omega_2 \) as follows:
\[ \Omega_1 = -\frac{Au_1' + x}{u_1' + x} \Omega_2 \]
\[ \sum \frac{\partial \theta}{\partial x} = \frac{\Omega_1 g'' + \Omega_2 g_1'}{|P|} \]
\[ = \frac{\Omega_2}{|P|} \left( g'' - \frac{Au_1' + x}{u_1' + x} g_2' \right) \]
\[ = \frac{\Omega_2}{|P|} \frac{x(g'' - g_2') + u_2'(g'' - Ag_2')}{u_2' + x} \]

Suppose

\[ |g''_2| > |g''_1|, \text{ i.e. the marginal cost increases at a higher rate in region 2} \]

Then

\[ g''_1 - g''_2 > 0 \text{ and } g''_1 - Ag''_2 > 0 \]

Together, these imply \( \sum \frac{\partial \theta}{\partial x} > 0 \)

**Proof of Lemma 5**

For linear quadratic case with similar utility,

\[ A_1 = -u_1'' \left( \frac{Au_2''(1 + \beta)}{Au_2'' + u_1''} \right)^2 = \frac{(\beta + 1)^2}{(1 + A)^2} \eta A^2 \]

\[ A_2 = -u_2'' \left( \frac{(1 + \beta)u_2''}{u_1' + Xu_2''} \right)^2 = \eta \left( \frac{(\beta + 1)^2}{(1 + A)^2} \right) \]

\[ SOC_1 = -\left( \delta_1 + A^2 \eta \frac{(\beta + 1)^2}{(1 + A)^2} \right) < 0 \]

\[ SOC_2 = -\left( \delta_2 + \frac{(\beta + 1)^2}{(1 + A)^2} \eta \right) < 0 \]

\[ \Omega_1 = -\frac{u_1''u_2''(1 + \beta)}{(Au_2'' + u_1'')^2} [Au_2' + x] \]
\[ = -\frac{(1 + \beta)}{(A + 1)^2} [Au_2' + 1] \]

\[ \Omega_2 = \frac{u_1''u_2''(1 + \beta)}{(Au_2'' + u_1'')^2} [u_2' + x] \]
\[ = \frac{(1 + \beta)}{(A + 1)^2} [u_2' + 1] \]

\[ \Omega_2 \left( SOC_1 \right) = -\frac{(\beta + 1)}{(A + 1)^2} (u_2' + 1) \left( \delta_1 + A^2 \eta \frac{(\beta + 1)^2}{(1 + A)^2} \right) < 0 \]

\[ \Omega_1 \left( SOC_2 \right) = \frac{(1 + \beta)}{(1 + A)^2} (u_2' + 1) \left( \delta_2 + \eta \frac{(\beta + 1)^2}{(1 + A)^2} \right) \]

\[ \Omega_2 \left( SOC_1 \right) \left( 1 + \frac{A_1}{SOC_1} \right) = -\frac{(1 + \beta)(u_2' + 1)}{(1 + A)^2} \left( \delta_1 + A^2 \eta \frac{(\beta + 1)^2}{(1 + A)^2} \right) \left( \frac{\delta_1}{\delta_1 + A^2 \eta \frac{(\beta + 1)^2}{(1 + A)^2}} \right) = \]
\[
\frac{(1 + \beta)(u'_2 + 1)\delta_1}{(1 + A)^2} - \frac{(1 + \beta)(u'_1 + 1)}{(1 + A)^2} (\delta_2 + \eta \frac{(\beta+1)^2}{(1+A)^2}) \left( \frac{\delta_2}{\delta_2 + \eta \frac{(\beta+1)^2}{(1+A)^2}} \right) =
\]
\[
\frac{(\beta + 1)(u'_1 + 1)}{(A + 1)^2} \delta_2
\]

Thus, the numerator
\[
\Omega_1(SOC_2) \left( 1 + \frac{A_1}{SOC_2} \right) + \Omega_2(SOC_1) \left( 1 + \frac{A_1}{SOC_1} \right) = \frac{(\beta + 1)(u'_1 + 1)}{(A + 1)^2} \delta_2 - \frac{(1 + \beta)(u'_2 + 1)}{(1 + A)^2} \delta_1
\]
\[
= \frac{(\beta + 1)(A u'_2 + 1)}{(A + 1)^2} \delta_2 - \frac{(1 + \beta)(u'_2 + 1)}{(1 + A)^2} \delta_1 = \frac{(1 + \beta)[(A \delta_2 - \delta_1) u'_2 + (\delta_2 - \delta_1)]}{(A + 1)^2}
\]
\[
\sum \frac{di}{dA} = \frac{(1 + \beta)}{|D|} \left( \frac{(A \delta_2 - \delta_1) u'_2 + (\delta_2 - \delta_1)}{(A + 1)^2} \right)
\]
Thus, if \( A \delta_2 < \delta_2 \leq \delta_1 \), \( \sum \frac{di}{dA} \leq 0 \). Otherwise, if \( \delta_2 > A \delta_2 > \delta_1 \), then
\[
\sum \frac{di}{dA} > 0.
\]

If the regions are symmetric such that \( \delta_2 = \delta_1 \), then \( \sum \frac{di}{dA} < 0 \).

**Proof of Lemma 6**

We have
\[
\frac{dP_1}{dA} = \frac{\partial P_1}{\partial \theta_1} \left( \sum \frac{di}{dA} \right) + \frac{\partial P_1}{dA}
\]
\[
\frac{\partial P_1}{\partial \theta_1} = \frac{A(1 + \beta)}{A + 1}
\]
\[
\sum \frac{di}{dA} = \frac{(1 + \beta)}{|D|} \left( \frac{(A \delta_2 - \delta_1) u'_2 + (\delta_2 - \delta_1)}{(A + 1)^2} \right)
\]
\[
\frac{\partial P_1}{dA} = \frac{(1 - \beta) \frac{u'_2}{A u'_2 + u'_1}}{(A + 1)^2} = -(1 - \beta) \frac{u'_2}{(A + 1)^2}
\]
\[
\frac{dP_1}{dA} = \frac{A(1 + \beta)}{A + 1} \frac{(1 + \beta)}{|D|} \left( \frac{(A \delta_2 - \delta_1) u'_2 + (\delta_2 - \delta_1)}{(A + 1)^2} \right) - (1 - \beta) \frac{u'_2}{(A + 1)^2}
\]
\[
= \frac{1}{(1 + A)|D|} \left( A(1 + \beta)^2 \frac{(A \delta_2 - \delta_1) u'_2 + (\delta_2 - \delta_1)}{(A + 1)^2} - (1 - \beta) \frac{|D|}{\eta} u'_2 \right)
\]
Now, \(|D| = (SOC_1) * (SOC_2) - A_1 A_2 \)
\[
= \left( \delta_1 + A^2 \eta \frac{(\beta+1)^2}{(1+A)^2} \right) \left( \delta_2 + \frac{(\beta+1)^2}{(1+A)^2} \right) - A^2 \eta \frac{(\beta+1)^2}{(1+A)^2} \eta \frac{(\beta+1)^2}{(1+A)^2}
\]
\[
= (\delta_1 + A^2 \delta_2) \eta \frac{(\beta+1)^2}{(1+A)^2} + \delta_1 \delta_2
\]

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Thus
\[
\frac{dP_1}{dA} = \frac{1}{(1+A)dD} \left( \frac{(A+\beta)^2 [(A\delta_2 - \delta_1)u'_2 + (\delta_2 - \delta_1)]}{(A + 1)^2} - (\delta_1 + A^2\delta_2) \frac{(\beta + 1)^2}{(1 + A)^2} - (1 - \beta) \frac{\delta_1\delta_2 u'_2}{\eta} \right)
\]
\[
= \frac{1}{(1+A)dD} \left( \frac{(\beta + 1)^2}{(1+A)^2} [(A\delta_2 - \delta_1)u'_2 + A\delta_2(1-A) - \delta_1(1+A)] - (1 - \beta) \frac{\delta_1\delta_2 u'_2}{\eta} \right)
\]
\[
\frac{dP_1}{dA} > 0 \text{ only if } A\delta_2 > \delta_1, A\delta_2(1-A) > \delta_1(1+A) \text{ and } \eta \text{ is sufficiently large.}
\]

If \( A \) is close to zero, then \( \frac{dP_1}{dA} < 0 \), i.e. public good consumption in the favourite province goes up as \( A \) decreases.

**Proof of Lemma 7**

Here, \( u''_i = -\eta, g''_i = -\delta \)

\[
\frac{dW}{dX} \bigg|_{A=1} = u'_2 \left[ \frac{\partial P_2}{\partial \theta_2} \frac{d\theta_2}{dA} + \frac{\partial P_2}{\partial \theta_1} \frac{d\theta_1}{dA} \right]
\]

\[
\frac{\partial P_2}{\partial \theta_2} \bigg|_{A=1} = \frac{(1+\beta)u'_2}{u''_2 A + u''_1} \bigg|_{A=1} = \frac{1}{2} (\beta + 1)
\]

\[
\frac{\partial P_1}{\partial \theta_1} \bigg|_{A=1} = \frac{(1+\beta)Au''_2}{u''_2 A + u''_1} \bigg|_{A=1} = \frac{1}{2} (\beta + 1)
\]

\[
\frac{d\theta_1}{dA} = \frac{\bigg[ \Omega_1 (SOC_2) + \Omega_2 A_1 \bigg]}{|D|}
\]

\[
\Omega_1 = -\frac{u''_2 (1+\beta)}{(Au''_2 + u''_1)^2} (Au'_2 + 1) \bigg|_{A=1} = -(u'_2 + 1) \left( \frac{1}{4} \beta + \frac{1}{4} \right)
\]

\[
\Omega_2 = \frac{u''_2 (1+\beta)}{(Au''_2 + u''_1)^2} (u'_2 + 1) \bigg|_{A=1} = (u'_2 + 1) \left( \frac{1}{4} \beta + \frac{1}{4} \right)
\]

Let \( \Omega_1 = -B, \Omega_2 = B \)

and \( k = \frac{\partial P_2}{\partial \theta_1} = \frac{1}{2} (\beta + 1) \).

\[
\frac{dW}{dA} \bigg|_{A=1} = u'_2 k \left[ \frac{d\theta_2}{dA} + \frac{d\theta_1}{dA} \right] \bigg|_{A=1}
\]

\[
\frac{d\theta_1}{dA} + \frac{d\theta_2}{dA} = \frac{(1+\beta)}{|D|} \left( \frac{(A\delta_2 - \delta_1)u'_2 + (\delta_2 - \delta_1)}{(A + 1)^2} \right)
\]

When \( A = 1 \),

\[
\frac{dW}{dA} \bigg|_{A=1} = \frac{k(1+\beta)u'_2}{4|D|} (\delta_2 - \delta_1) (1 + u'_2) < 0 \text{ iff } \delta_2 < \delta_1
\]

On the other hand,

\[
\frac{dW}{dA} \bigg|_{A=0} = u'_2 \left[ -\frac{\partial P_1}{\partial A} + \frac{\partial P_2}{\partial \theta_2} \frac{d\theta_2}{dA} \right] > 0
\]
Bibliography


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