Cool Models of Business Cycles
EC6012 2009 Lecture 3

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Today

1. Introduction
2. Data
3. Multiplier-Accelerators
4. Goodwin’s Growth Model
5. Minsky
   - Generating a crisis
6. Summary
Introduction
Stuff you’ll learn today

- You’ll see some BC data
- You’ll see 4 cool models
  1. Samuelson’s Multiplier Accelerator
  2. Goodwin’s Nonlinear Accelerator
  3. Goodwin’s Growth Cycle
  4. Minsky’s Financial Fragility Model
  5. Obstfeld & Rogoff’s Redux (time allowing)
- I’ll show you why they are cool as we go.
Ireland’s Real Output has been all over the place since 1970

Figure: Ireland’s Year on Year Percentage Real GDP Growth, 1970-2007.
But Look at Fixed Investment

Figure: Logged Fixed Investment in Ireland, 1970–2008.
Government Consumption (logs)

1980 1990 2000

Figure: Logged Government consumption in Ireland, 1970–2008.
Inventory Change (logs)

Figure: Logged changes in inventory for Ireland, 1970–2008. Missing data is a reporting error.
Figure: Logged total consumption in the Irish economy, 1970-2008.
Different Explanations for these changes.
Real Business Cycle People Say

(2, pg. 1):

\[ t \text{he economy is viewed as being in continuous equilibrium in the sense that, given the information available, people make decisions that appear optimal for them, and so do not make persistent mistakes. This is also the sense in which behaviour is said to be rational. Errors, when the occur, are said to be information gaps, such as unanticipated shocks to the economy.} \]
Different Explanations for these changes.
Non mainstream People Say

(1, pg. 199):

The financing of investment by means of new techniques means the generation of demand in excess of that allowed for by the existing tranquil state. The rise in spending upon investment leads to an increase in profits, which feeds back and raises the price of capital assets and thus the demand price of investment. Thus, any full-employment equilibrium leads to an expansion of debt-financing—weak at first because of the memory of preceding financial difficulties—that moves the economy to expand beyond full employment. Full employment is a transitory state because speculation upon and experimentation with liability structures and novel financial assets will lead the economy to an investment boom. An investment boom leads to inflation, and, by processes still to be described, an inflationary boom leads to a financial structure that is conducive to financial crises.
Multiplier Accelerators

\[ Y = C + I + G + X - M, \]  \hfill (1)

[Which Says] National output \((Y)\) is the sum of Consumption, \(C\), Investment, \(I\), Government expenditure, \(G\), and exports minus imports, \(X - M\).
Multiplier-Accelerators

Consumption

\[ Y_t = c_0 + c_1 Y_{t-1}. \]  \hspace{1cm} (2)
$I_t = I_0 + I(r) + b(C_t - C_{t-1})$.  \hfill (3)

**Caution**

We need $b > 0$. We can assume a constant interest rate pretty easily, which is the same thing as saying let’s drop it entirely. Assuming $I(r) = 0...$
Multiplier-Accelerators

\[ l_t = l_0 + b(C_t - C_{t-1}). \]  \hspace{1cm} (4)

AD Becomes

\[ Y_{t}^d = C_t + l_t = c_0 + l_0 + cY_{t-1} + b(C_t - C + t - 1). \]  \hspace{1cm} (5)

Assume \( Y^d = Y \) at time \( t \)
Multiplier-Accelerators

\[ Y_t = c_0 + l_0 + cY_{t-1} + b(C_t - C_{t-1}) \] (6)
Know values of $C_t$ and $C_{t-1}$ are $C_t = c_0 + cY_{t-1}$, and $C_{t-1} = c_0 + cY_{t-2}$. Subbing these in:

$$Y_t = c_0 + l_0 + cY_{t-1} + b(c_0 + cY_{t-1} - c_0 - cY_{t-2})$$

(7)

Write equation 7 as a second order linear difference equation:

$$Y_t - (1 + b)cY_{t-1} + bcY_{t-2} = (c_0 + l_0)$$

(8)
Solution

We solve difference equations by finding their equilibrium or steady states, where $Y_t = Y_{t-1} = Y_{t-2} = Y^*$. Putting this into equation 8 and rearranging, we get

$$Y^* = \frac{c_0 + l_0}{1 - c}$$

(9)
Lessons from Samuelson

1. Cycle theory is tricky, normally requires a bit of differential calculus;
2. Some cycles are more ‘cyclical’ than others. Some will explode, some will dampen, some will oscillate, some will focus in on one point. This is where phase plots and arrow diagrams come in really handy;
3. The economy is highly dependent on past values of itself for its current levels of output, employment, etc., so lagged effects are always going to matter in these models. (ARMA/ARIMA/etc modeling)
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Non Linear Accelerator

Call capital stock \( k \), \( \psi \) is the desired capital stock proportional to income or output, \( C \) is consumption, \( y \) is income, \( c_0, c_1 \) and \( b \) are constants. Assuming a linear consumption function which relates consumer spending, \( C \), to income, \( Y \), such as \( c = c_0 + c_1 Y \delta \), we have

\[
\psi = by, \quad (10)
\]

\[
C = c_0 + c_1 y, \quad (11)
\]

\[
y = C + \dot{k}, \quad (12)
\]

Buildup: \( k^* \); scrapping rate \( k^{**} \)

\[
\dot{k} = \begin{cases} 
k^*, & \psi > k, \\
0, & \psi = k, \\
k^{**}, & \psi < k. \end{cases} \quad (13)
\]
Nearly there...

Now combine equations 10, 11, 12 and 13, to obtain

$$\psi = \frac{b}{1 - c_1} \dot{k} + \frac{c_0 b}{1 - c_1}.$$  \hspace{1cm} (14)
Figure: Phase Diagram of the Non Linear Multiplier/Accelerator.
Very Cool
Why?

Even though it is really simple, this model is cool for at least four reasons.

1. The final result is independent of initial conditions;
2. The oscillation maintains itself without any stochastic shocks whatsoever;
3. The equilibrium exists, is attainable, but is unstable, which makes sense to us intuitively;
4. No lags are required for this model to work, unlike Samuelson’s.
Setup

- Predator Prey Interaction
- Model is coupled equations where the interesting dynamics came from feedbacks and interactions
- Two homogeneous, non-specific factors of production, labour, $L$ and capital, $K$, where all quantities are real and net, and all wages are consumed with all profits being reinvested into the system.
- A steady growth rate $\beta$ of the labour force $N$ according to $N = N_0 e^{\beta t}$
- Steady technical progress, $\alpha$ so that the capital-labour ratio evolves according to $y/l = \alpha = \alpha_0 e^{\alpha t}$
- The capital-output ratio $k = Y/L$ is assumed constant and the real wage rises in the neighbourhood of full employment
- The workers accrue to themselves a portion of the output of the economy, $u$ and the capitalists receive $v$ for their efforts
\[ \dot{v} = \left\{ \left[ \frac{1}{k} - (\alpha + \beta) \right] - \frac{1}{k} u \right\} v \]  

(15)

\[ \dot{u} = -[(\alpha + \gamma) + \rho v] u \]  

(16)
Goodwin on his cycle:

When profit is greatest, $u = u$, employment is average, . . . , and the high growth rate pushes employment to its maximum $v_2$, which squeezes the profit rate to its average value . . . the deceleration in the growth employment (relative) to its average value again, where profit and growth are again at their nadir $u_2$. This low growth rate leads to a fall in output and employment to well below full employment, thus restoring profitability to its average value because productivity is now rising faster than wage rates . . . . The improved profitability carries the seed of its own destruction by engendering a too vigorous expansion of output and employment, thus destroying the reserve army of labour and strengthening labour’s bargaining power.
Figure: Evolution of capitalist/Worker interactions as they share the products of the economy. We see here that the motion is cyclical and bounded, implying the dynamics of the system exhibit limit cycle behaviour.
Why is this cool?

This is a cool model for three reasons.

1. The model generates a feedback-driven limit cycle, showing workers dependent on capitalists and vice versa.
2. It shows Marxian macrodynamics in an interesting light.
3. The model can be extended to include search and selection, endogenising the values of the parameters used in the model, see www.stephenkinsella.net/research for details.
Minsky Setup

- Debt structure of firms matters.
- Market is naturally unstable
- Presence of Big Bank and Big Government dampen cycles
- Booms and busts are the inevitable result of institutionally-legitimised high risk lending practices
Model

A constant markup $\tau$ over the wage bill $w$, and the labour/output ratio is $b$. The price level $P$ is determined by

$$P = (1 + \tau)wb.$$  \hspace{1cm} (17)

The profit rate, $r$, is given by adding up the contributions to profit from the various sectors of the economy:

$$r = \frac{PX - wbX}{PK} = \frac{\tau wbX}{(1 + \tau)wbX} = \frac{\tau}{1 + \tau} \frac{X}{K},$$  \hspace{1cm} (18)

Big Idea

The core of Minsky’s theory revolves around how expected returns relate to the capital stock, $K$. 
Investment Decision

\[ P_k = (r + \rho)P/i, \]  \hspace{1cm} (19)

\[ P_k - P = (r + \rho - i)P/i. \]  \hspace{1cm} (20)

Investment Demand = \( PL = [g_0 + h(r + \rho - i)]PK. \)  \hspace{1cm} (21)

Saving Supply = \( srPK = s\tau wbX. \)  \hspace{1cm} (22)
Equilibrium Conditions

\[ g_0 + h(r + \rho - i) - sr = 0. \]  
(23)

Solve equation 23 for \( r \), plug it into the investment demand function, and we get an expression for the capital stock growth rate, \( g = l/K \).

\[ g = s[g_0 + h(\rho - i)]s - h. \]  
(24)

Cool equation

Equation 24 is very cool: a fall in the interest rate, or an increase in anticipated profits leads to higher growth, since \( g = sr \) from the saving function, so the profit rate and capacity utilization go up as well.
Financial Side of the Economy

Fiscal debt: $F$. Can be converted into money, $M$, or short term bonds $B$, bond is held by rentiers. Value of all plant and equipment is $P_k K = (r + \rho) PK / i$. Firms have equity, $E$, which has a market price at $P_e$. The difference between capital stock and equity is firms’ net worth, $N$. The differential of the firms’ balance sheets is

$$P_k I + \dot{P}_k = \dot{P}_k K = P_e \dot{E} + \dot{P}_e E + \dot{N}. \quad (25)$$
The total wealth of all rentiers is

\[ W = P_e E + M + B = P_e E + F. \]  \hfill (26)

The rentiers’ wealth changes over time according to

\[ \dot{W} = \dot{P}_e E + P_e \dot{E} + \dot{M} + \dot{B} = \dot{P}_e E + srPK. \]  \hfill (27)

Which says...

Rentiers get rich from increases in capital gains and financial saving.
At each point in time, rentiers have to decide to allocate their wealth across assets according to these balancing rules:

\[
\mu(i, r + \rho)W = M = 0, \quad (28)
\]

\[
\frac{\epsilon(i, r + \rho)}{P_e}W - E = 0, \quad (29)
\]

\[
- \beta(i, r + \rho)W + B = 0. \quad (30)
\]

Here \(\mu + \epsilon + \beta = 1\). The asset demand equations given above determine the interest rate and the anticipated rate of profit on physical capital, \(r + \rho\).
We can think of $r + \rho$ as representing returns to equity. Higher returns will bid up the value of firm’s capital stock in this economy.

Combining 26 and 29, we have

$$W = \frac{F}{1 - \epsilon(i, r + \rho)}.$$  \hfill (31)

Equation 31 says that increasing $r$ or $\rho$ will drive up $\epsilon$, and so share prices and financial prices will rise.
Macro policies determine micro net worth

Rentier’s net worth is determined macroeconomically by their valuation of anticipated profits, which feeds demand for asset supplies and demands in the current period.

\[ P_e = \left( \frac{\epsilon}{1 - \epsilon} \right) \left( \frac{F}{E} \right); \]  

(32)

In turn \( P_e \) will determine the changes in firms’ net worth, given their investment levels and issuance of new equity, and excess demand in the money markets will be the sum of

\[ \mu(i, r + \rho) = \frac{M}{F} \left[ 1 - \epsilon(i, r + \rho) \right], \]  

(33)
Equations 33 and 24 pick out an ISLM relation which looks like this:

Figure: Response of Interest and Profit rates to an increase in expected profit rate $\rho$. 
Adjustment Dynamics

Let the change in expected profits be given by

$$\dot{\rho} = -\beta(i - \bar{i}).$$ \hspace{1cm} (34)

When the rate of interest exceeds its normal long run level, $\bar{i}$, expected profits will begin to fall, and fall sharply.
Call the money debt ratio $\alpha$, and write

$$\alpha = \frac{M}{F} = \frac{M}{PK} \frac{PK}{F} = \frac{M}{PK} \left( \frac{1}{f} \right),$$

(35)

where $f$ is the ratio of outstanding fiscal debt to the capital stock. Fix government expenditure as a proportion of the capital stock (and taxes of expenditures). Now $f$ is fixed. The money debt ratio evolves according to

$$\hat{\alpha} = \hat{M} - g.$$

(36)

This says as money grows, $\hat{\alpha}$ falls as $g$ increases.
Stability Check

We can check for stability close to an equilibrium point \((i = \bar{i}, g = \hat{M})\) using the Jacobian:

\[
\begin{bmatrix}
-\beta i_{\rho} & -\beta i_{\alpha} \\
-(g i_{\rho} + g_{\rho}) & -g i_{\alpha}
\end{bmatrix}
\]  

(37)

Idea

Any increase in \(\rho\), investor confidence, will lower the interest rate, and raise the derivative of \(\rho\) in equation 34. This is positive feedback. There can of course be negative feedback. Crises can come at any moment.
Adjustment Dynamics

$$\text{Anticipated incremental profit rate } \rho = 0$$

$$\text{Ratio of money to outside assets } a$$

**Figure:** Adjustment Dynamics
Debt Deflation Dynamics

The economy starts at point A. Any drop in investor confidence will move it to point B, where authorities will try, through policy, to increase $M$ and hence the money debt ratio, $\hat{\alpha}$. This would move the economy to point C, and back to equilibrium. If the economy does not turn the corner at C, then the economy enters a debt-deflation scenario.
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