Models of Money Demand & Theories of Interest Rate Determination
International Monetary Economics, Lecture 7

Stephen Kinsella*

March 16, 2009

1 Introduction

Last week we saw three functions central banks normally carry out in the stabilisation of economies: they have control over the money supply, which allows them to affect exchange rates, they have control over deposits, and so can decide what proportion of reserves private banks have to hand over to them in exchange for a license to bank. This is the reserve ratio. Finally, central banks can set the interest rate on discount loans to brokers and private banks. If they drop the discount rate, central banks, acting through private banks, allow lending to increase, and thus real economic activity to increase Leddin and Walsh [2003], Blanchard [2005]. So the story goes.

In this lecture, we'll look at models of money demand, interest rate determination, seigniorage, and inflation. We'll see these phenomena are actually fairly tightly interwoven, and the models we'll use to explain them are easily built, derived, and extended—in the case of exogenous money, of course. When we introduce endogenous money, things will get hairy.

2 Money demand models, deficits, seigniorage, and inflation

From [Blanchard and Fischer, 1998, pp. 512], let's build a simple model of money demand. Initially let's assume money demand is determined solely by the demand and supply of a fiat currency, so we're implicitly living in a world where the price for and quantity of money is determined by simple supply and demand.

Most of the time, money demand models are used to study *inflation*, the persistent (positive) change in the price level in the economy. Several authors, notably Sargent and Ljungqvist [2004], find that persistent budget deficits are correlated with persistent inflation stabilisation. This is because fiscal reforms (normally increased income and consumption taxes, reduced public expenditure, etc) tend to dent domestic consumption, which make nominal price appreciation hard in any non-monopolistic industry or sector.

The counter argument to people like Sargent is that deficits tend to be associated with differing expectations of the growth rate of the money stock. When there is a large deficit and the stock

*Department of Economics, Kemmy Business School, University of Limerick, Ireland. Email: stephen.kinsella@ul.ie, www.stephenkinsella.net

of money is high and growing, government revenue from printing money—seigniorage—is also correspondingly high. The public, knowing this, will anticipate higher rates of money growth (a hyperinflation), or increased taxes later on, and this will change their behaviour today. The public will either substitute away from using the currency altogether, or reduce spending, which will curb inflation rates. The argument comes from **?** but had been elaborated upon for countries like Pakistan and Mexico.

2.1 Model

In this economy money demand is given by

$$\frac{M}{P \times Y} = f(r + \pi^*) \tag{1}$$

Here Y is output, r is the real interest rate, π^* is the expected rate of inflation. Hold Y and r constant. Money demand goes down when the nominal interest rate increases, ($f'() < 0$), and when actual inflation, π_A, is such that $\pi_A > \pi^*$, money demand goes up, and when $\pi_A < \pi^*$, money demand goes down.

Let δ be the level of the primary deficit the government is planning on running as a percentage of GNP. There will be a primary deficit, δ_0, and this should be positive, so we'll have

$$\delta = \delta_0 + rb \tag{2}$$

where b is the ratio of government bonds to GNP.

The government will finance its debt through money creation and issuance of bonds. It will finance some share of the deficit, α, by printing money, and the rest by borrowing, $(1 - \alpha)$.

Therefore

$$\frac{dM/dt}{PY} = \alpha\delta \tag{3}$$

and

$$\frac{db}{dt} = (1 - \alpha)\delta \tag{4}$$

Substitute 2 into equation 4 to get

$$\frac{db}{dt} = (1 - \alpha)[\delta_0 + rb] \tag{5}$$

This says financing of the government through borrowing can't last. Fiscal reforms could be 1. 100% money financing, or 2. increased taxes and/or more seigniorage.

What are the inflation dynamics for both scenarios?

2.2 Money Financing

Differentiate 1 with respect to time, assume $\pi = \pi^*$, and you'll get

$$f'(r + \pi)(d\pi/dt) = \underbrace{\alpha(\delta_0 + rb)}_{\text{Seiniorage}} - \underbrace{}\pi \times f(r + \pi)_{\text{Inflation Tax}} \tag{6}$$

Equations 6 and 5 map out a 2x2 system of difference equations, which I'll plot in (π, b) space in class.

- In the phase space, at $d\pi/dt = 0$, the level of seniorage will equal the inflation tax
- At low levels of inflation, a jump in inflation will lead to a high inflation tax, implying a higher feasible level of debt
- At $\pi^{'} = \pi$, seigniorage is maximised
- At high levels of inflation, seigniorage declines, and so does the level of 'sustainable' debt.
- At $db/dt = 0$ the government is a net creditor.

The dynamics of the system can be seen if we look at the equations of motion. At $db/dt = 0$, debt increases above $\delta = 0$, and decreases below $\delta = 0$.

Inflation is increasing above $d\pi/dt = 0$ and decreasing above $d\pi/dt = 0$. There is no saddle point to this system, and in fact, there will only be convergence if at the start of the system, $\pi = \delta = 0$.

We can calculate the value of debt in the system by integrating up from t to T, and call this total debt amount b_T. Now compute an inflation rate you'd expect if you always had this amount of debt financed by money alone, and we'll get the dynamics of inflation, where

$$\pi f(r + \pi) = \delta_0 + rb_T. \tag{7}$$

This equation has a locus at $d\pi/dt = 0$ and $\delta = 0$, and we can now talk about expectations. If we start with low inflationary expectations, we stay on the lower bound. If we start with high expectations, we immediately jump to the higher debt/inflation tradeoff curve.

3 A model with endogenous money demand

Now we are moving on, to extend the stock flow consistent SIM in interesting and more realistic ways. The first and most reasonable place to begin is by adding a **central** bank which can issue T-bills. These bills will give an interest rate, r, and have the same price over their lifetimes to simplify the analysis. We'll call this model PC for portfolio choice. We'll see that we immediately need to endogenise money creation to close the model, as well as breaking the household's decisions up into two stages: first, a consumption/saving decision, and second, an allocation decision. These take place in the same period, but sequentially. Capital is still instantaneously created and destroyed (a haircut economy), and there is still no production as we are still in a pure service economy, so things are pretty simple in the balance sheets, as we see in table 1:

	Households	Production	Government	Central Bank	$\sum$
Money	$+H$			$-H$	0
Bills	$+B_h$		$-B$	$+B_{cb}$	0
Balance (net worth)	-V		$+V$		0
$\sum$	0		0	0	0

Table 1: Balance Sheet for PC.

The sum of household wealth is now V, where the wealth is a sum of household holdings of cash (H), and bonds (B_h). Private wealth has got to be equal to public debt in this system, so we see in the Balance of table 1 that $-V$ occurs in the household's balance sheets, and $+V$ in the central bank's.

The transactions flows for the economy is given by table 2:

				Central Bank		
	Households	Production	Government	Current	Capital	$\sum$
Consumption	- C	+ C				0
Govt. Expenditures		+ G		-G		0
Income = GDP	+Y	-Y				0
Interest Payments	$-r_{-1} \cdot B_{h-1}$		$+r_{-1} \cdot B_{-1}$	$+r_{-1} \cdot B_{cb-1}$		0
Central Bank Profits			$+r_{-1} \cdot B_{cb-1}$	$-r_{-1} \cdot B_{cb-1}$		0
Taxes	-T	+T				0
Change in Money	$-\Delta H$				$+\Delta H$	0
Change in Bills	$-\Delta B_H$	$+\Delta B$			$-\Delta B_{cb}$	0
$\sum$	0	0	0	0	0	0

Table 2: Transactions matrix for PC.

Here again, all rows and columns sum to zero, so all transactions are taken into account, but this time we have to take account of interest payments arising from stocks of assets issued by the Central Bank, and the flow of funds to and from households has these financial assets in them (money and bonds). The really big change, though, is the introduction of a central bank 'sector' in the economy. This has two sections: capital and current, and this sets us up for chapter 6, when we introduce and open economy model.

3.1 Equation System

$$Y = G + C \tag{8}$$

$$YD = Y - T + r_{-1} \cdot B_{h-1} \tag{9}$$

$$T = \theta \cdot (Y + r_{-1} \cdot B_{h-1}) \tag{10}$$

$$V = V_{-1} + (YD - C) \tag{11}$$

$$C = \alpha_1 \cdot YD + \alpha_2 \cdot V_{-1}, 0 < \alpha_1 < \alpha_2 < 1 \tag{12}$$

$$\frac{H_h}{V} = (1 - \lambda_0) - \lambda_1 \cdot r + \lambda_2 \cdot \left(\frac{YD}{V}\right) \tag{13}$$

$$\frac{B_h}{V} = \lambda_0 + \lambda_1 \cdot r - \lambda_2 \cdot \left(\frac{YD}{V}\right) \tag{14}$$

$$H_h = V - B_h \tag{15}$$

$$\Delta B_s = B_s - B_{s-1} = (G + r_{-1} \cdot B_{s-1}) - (T + r_{-1} \cdot B_{cb-1}) \tag{16}$$

$$\Delta H_s = H_s - H_{s-1} = \Delta B_{cb} \tag{17}$$

$$B_{cb} = B_s - B_h \tag{18}$$

$$r = \overline{r} \tag{19}$$

3.2 Steady State Solutions

$$\alpha_3 = \alpha_2 \cdot (1 - \alpha_1)/\alpha_2 \quad (20)$$
$$\Delta V = \alpha_2 \cdot (\alpha_3 - V_{-1}) \quad (21)$$
$$\frac{V^*}{YD^*} = \alpha_3 \quad (22)$$
$$r^* = \frac{B_h^* \cdot r}{V^*} \quad (23)$$

4 PCEX, Portfolio Choice with Expectations

Introducing expectations into PC is done through including an expectation on disposable income, YD^e. This changes the consumption function to

$$C = \alpha_1 \cdot YD^e + \alpha_2 \cdot V_{-1}. \quad (24)$$

How does this affect the consumption function? Well, households now don't know exactly what they will consume, so they can't perfectly forecast their end of period incomes, or their end of period holdings of bills, so the liquidity preference relations have to be rewritten to show assets held at the start of the period (d), and the end (h). We require two new liquidity preference equations, and an adding up constraint which balances the funds out at the end of each period.

$$\frac{B_d}{V^e} = \lambda_0 + \lambda_1 \cdot r - \lambda_2 \cdot \left(\frac{YD^e}{V^e}\right) \quad (25)$$
$$\frac{H_d}{V^e} = (1 - \lambda_0) - \lambda_1 \cdot r + \lambda_2 \cdot \left(\frac{YD^e}{V^e}\right) \quad (26)$$
$$H_d = V^e - B_d \quad (27)$$
$$V^e = V_{-1} + (YD^e - C) \quad (28)$$

4.1 Puzzling Results from the Model

- $\uparrow G \Rightarrow \uparrow YD$, $\frac{\partial YD^*}{\partial G} < 0$
- $\uparrow r \Rightarrow \uparrow YD$, $\frac{\partial YD^*}{\partial r} > 0$
- $\uparrow \lambda_0 \Rightarrow$ Govt. taking Bills $\Rightarrow \uparrow Y$. So dropping liquidity preference implies increasing Y in PCEX.
- $\uparrow \alpha_3 \Rightarrow \uparrow Y$ (Paradox of Thrift?)

4.2 Real World Applications

PCEX implies that targeting debt to income ratios will have a positive effect on the fortunes of the country, if the government does the targeting in a credible way. In particular, PCEX gives us a targeting rule of

$$\frac{V^*}{Y^*} = \frac{\alpha_3}{1 + \left[\frac{\theta}{1-\theta}\right] \cdot r \cdot [(\lambda_0 + \lambda_1 \cdot r) \cdot \alpha_3 - \lambda_2]}. \tag{29}$$

The government controls G, θ and r, so it can control the level of Y^* once it knows α_3. The Irish government has a set of policies around controlling these three parameters. What are they called, and have these policies worked? Figures 1 and 2 will tell the story.

PCEX also implies that knowing α_3 is important in forecasting behaviour. What does today's news about the percentage of SSIA money actually spent imply about the level of α_3 in the Irish economy at present?

Notation

Symbol	Meaning
G	Pure government expenditures in nominal terms
Y	National Income in Nominal Terms
C	Consumption of goods supply by households, in nominal terms
T	Taxes
θ	Personal Income Tax Rate
YD	Disposable Income of Households
α_1	Propensity to consume out of regular (present) income
α_2	Propensity to consume out of past wealth
ΔH_s	Change in cash money supplied by the central bank
ΔH_h	Cash money held by households
H, H_{-1}	High Powered cash money today, and yesterday ($_{-1}$)
V	Wealth of Households, in nominal terms
$B_{h,cb}$	Bills held by households, central banks.

References

Olivier Blanchard. *Macroeconomics*. Prentice Hall, 4rth edition, 2005.

Olivier Blanchard and Stanley Fischer. *Lectures on Macroeconomics*. MIT Press, Cambridge, MA, 1998.

Anthony J. Leddin and Brendan M. Walsh. *The Macroeconomy of the Eurozone: An Irish Perspective*. Gill and Macmillan, 1st edition, 2003. URL http://www.gillmacmillan.ie/Ecom/Library3.nsf/CatalogByCategory/DA55B901FD6459BB80256C1C003FEE1F?OpenDocument.

T. Sargent and L. Ljungqvist. *Recursive Macroeconomic Theory*. MIT Press, 2004.