EC6012 Lecture 7

Models of Money Demand

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Last Time

Central Banks control

- Reserve Ratio Leddin and Walsh (2003)
- Discount Rate Blanchard (2005)
- Money Supply, M0, M1, etc. Abel et al. (2007)

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Today: What about Money Demand?

Two Models, one with Exogenous MD, one with Endogenous.

Simple Model

Due to Sargent and Ljungqvist (2004) and **?**, (Blanchard and Fischer, 1998, pp. 512).

- Idea: Assume money demand is determined solely by the demand and supply of a fiat currency.
- Want to explain why persistent budget deficits are correlated with persistent inflation stabilisation Sargent and Ljungqvist (2004)
- Really useful model for thinking about fiscal reforms
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Model

In this economy money demand is given by

$$\frac{M}{P \times Y} = f(r + \pi^*) \tag{1}$$

Here Y is output, r is the real interest rate, π^* is the expected rate of inflation. Hold Y and r constant. Money demand goes down when the nominal interest rate increases, ($f'() < 0$), and when actual inflation, π_A, is such that $\pi_A > \pi^*$, money demand goes up, and when $\pi_A < \pi^*$, money demand goes down.

Thinking about Deficits

Let δ be the level of the primary deficit the government is planning on running as a percentage of GNP. There will be a primary deficit, δ_0, and this should be positive, so we'll have

$$\delta = \delta_0 + rb \tag{2}$$

where b is the ratio of government bonds to GNP.
The government will finance its debt through money creation and issuance of bonds. It will finance some share of the deficit, α, by printing money, and the rest by borrowing, $(1 - \alpha)$.

Relationships

Therefore

$$\frac{dM/dt}{PY} = \alpha\delta \tag{3}$$

and

$$\frac{db}{dt} = (1-\alpha)\delta \tag{4}$$

Mucking around with Algebra

Substitute 2 into equation 4 to get

$$\frac{db}{dt} = (1 - \alpha)[\delta_0 + rb] \quad (5)$$

This says financing of the government through borrowing can't last. Fiscal reforms could be 1. 100% money financing, or 2. increased taxes and/or more seigniorage.
What are the inflation dynamics for both scenarios?

Money Financing

Differentiate 1 with respect to time, assume $\pi = \pi^*$, and you'll get

$$f'(r + \pi)(d\pi/dt) = \alpha(\delta_0 + rb) - \pi \times f(r + \pi) \tag{6}$$

Equations 6 and 5 map out a 2x2 system of difference equations, which I'll plot in (π, b) space now.

Phase Space Plot

Description of the Space

- In the phase space, at $d\pi/dt = 0$, the level of seniorage will equal the inflation tax
- At low levels of inflation, a jump in inflation will lead to a high inflation tax, implying a higher feasible level of debt
- At $\pi^{'} = \pi$, seigniorage is maximised
- At high levels of inflation, seigniorage declines, and so does the level of 'sustainable' debt.
- At $db/dt = 0$ the government is a net creditor.

Dynamics

The dynamics of the system can be seen if we look at the equations of motion. At $db/dt = 0$, debt increases above $\delta = 0$, and decreases below $\delta = 0$.
Inflation is increasing above $d\pi/dt = 0$ and decreasing above $d\pi/dt = 0$.
There is no saddle point to this system, and in fact, there will only be convergence if at the start of the system, $\pi = \delta = 0$.
We can calculate the value of debt in the system by integrating up from t to T, and call this total debt amount b_T. Now compute an inflation rate you'd expect if you always had this amount of debt financed by money alone, and we'll get the dynamics of inflation, where

$$\pi f(r + \pi) = \delta_0 + rb_T. \tag{7}$$

This equation has a locus at $d\pi/dt = 0$ and $\delta = 0$, and we can now talk about expectations. If we start with low inflationary expectations, we stay on the lower bound. If we start with high expectations, we immediately jump to the higher debt/inflation tradeoff curve.

Now for something completely different

Notation

Symbol	Meaning
G	Pure government expenditures in nominal terms
Y	National Income in Nominal Terms
C	Consumption of goods supply by households, in nominal terms
T	Taxes
θ	Personal Income Tax Rate
YD	Disposable Income of Households
α_1	Propensity to consume out of regular (present) income
α_2	Propensity to consume out of past wealth
ΔH_s	Change in cash money supplied by the central bank
ΔH_h	Cash money held by households
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Review

	Households	Production	Government	Central Bank	$\sum$
Money	$+H$			$-H$	0
Bills	$+B_h$		$-B$	$+B_{cb}$	0
Balance (net worth)	-V		$+V$		0
$\sum$	0		0	0	0

Table: Balance Sheet for PC.

Transactions Matrix

				Central Bank		
	Households	Production	Government	Current	Capital	$\sum$
Consumption	- C	+ C				0
Govt. Expenditures		+ G	-G			0
Income = GDP	+Y	-Y				0
Interest Payments	$+r_{-1} \cdot B_{h-1}$		$-r_{-1} \cdot B_{-1}$	$+r_{-1} \cdot B_{cb-1}$		0
Central Bank Profits			$+r_{-1} \cdot B_{cb-1}$	$-r_{-1} \cdot B_{cb-1}$		0
Taxes	-T		+T			0
Change in Money	$-\Delta H$				$+\Delta H$	0
Change in Bills	$-\Delta B_h$		$+\Delta B$		$-\Delta B_{cb}$	0
$\sum$	0	0	0	0	0	0

Table: Transactions matrix for PC.

Equation Systems

$$Y = G + C \tag{8}$$

$$YD = Y - T + r_{-1} \cdot B_{h-1} \tag{9}$$

$$T = \theta \cdot (Y + r_{-1} \cdot B_{h-1}) \tag{10}$$

$$V = V_{-1} + (YD - C) \tag{11}$$

$$C = \alpha_1 \cdot YD + \alpha_2 \cdot V_{-1}, 0 < \alpha_1 < \alpha_2 < 1 \tag{12}$$

$$\frac{H_h}{V} = (1 - \lambda_0) - \lambda_1 \cdot r + \lambda_2 \cdot \left(\frac{YD}{V}\right) \tag{13}$$

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\Delta H_s &= H_s - H_{s-1} = \Delta B_{cb} && (17)\\
B_{cb} &= B_s - B_h && (18)\\
r &= \bar{r} && (19)
\end{aligned}$$

Steady States

$$\alpha_3 = \alpha_2 \cdot (1 - \alpha_1)/\alpha_2 \quad (20)$$

$$\Delta V = \alpha_2 \cdot (\alpha_3 - V_{-1}) \quad (21)$$

$$\frac{V^*}{YD^*} = \alpha_3 \quad (22)$$

$$r^* = \frac{B_h^* \cdot r}{V^*} \quad (23)$$

PCEX

Introducing expectations into PC is done through including an expectation on disposable income, YD^e. This changes the consumption function to

$$C = \alpha_1 \cdot YD^e + \alpha_2 \cdot V_{-1}. \quad (24)$$

Expectation-Augmented Modeling

$$\frac{B_d}{V^e} = \lambda_0 + \lambda_1 \cdot r - \lambda_2 \cdot \left(\frac{YD^e}{V^e}\right) \quad (25)$$

$$\frac{H_d}{V^e} = (1 - \lambda_0) - \lambda_1 \cdot r + \lambda_2 \cdot \left(\frac{YD^e}{V^e}\right) \quad (26)$$

$$H_d = V^e - B_d \quad (27)$$

$$V^e = V_{-1} + (YD^e - C) \quad (28)$$

Puzzling Results from the Model

- $\uparrow G \Rightarrow \uparrow YD$, $\frac{\partial YD^*}{\partial G} < 0$
- $\uparrow r \Rightarrow \uparrow YD$, $\frac{\partial YD^*}{\partial r} > 0$
- $\uparrow \lambda_0 \Rightarrow$ Govt. taking Bills $\Rightarrow \uparrow Y$. So dropping liquidity preference implies increasing Y in PCEX.
- $\uparrow \alpha_3 \Rightarrow \uparrow Y$ (Paradox of Thrift?)

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Real World Applications

Application 1

PCEX implies that targeting debt to income ratios will have a positive effect on the fortunes of the country, if the government does the targeting in a credible way. In particular, PCEX gives us a targeting rule of

$$\frac{V^*}{Y^*} = \frac{\alpha_3}{1 + \left[\frac{\theta}{1-\theta}\right] \cdot r \cdot [(\lambda_0 + \lambda_1 \cdot r) \cdot \alpha_3 - \lambda_2]}. \quad (29)$$

Next Time

- Presentations $\longrightarrow$ Check Site.
- Read Godley, Chapter 6
- Read Expectations In Economics by Lachman (Link on Site, be in college)
- The Open Economy!

References

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