Inflationary Consequences of Anticipated Macroeconomic Policies

ALLAN DRAZEN and ELHANAN HELPMAN
Tel-Aviv University

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Budget deficits implying an unbounded present value of government debt are infeasible and hence induce expectations of a future policy change. We study how expectations of a policy switch whose timing or mix between expenditure cuts, tax increases or increases in money growth rates may be uncertain, affect economic dynamics before the switch takes place. We are especially concerned with the correlation between changes in the deficit and inflation. Of particular interest is our finding that timing uncertainty may induce fluctuations in the rate of inflation that seem to be unrelated to the budget deficit, at a time when the budget deficit is responsible for inflation.

1. INTRODUCTION

What happens when the government follows monetary and fiscal policies which are known to be infeasible over the long run? The recognized unsustainability of policy means that it is known that at some point in the future the current policy will be abandoned. What is generally not known is exactly when the policy switch will take place and what the new policy mix will be. Our purpose in this paper is to study how expectations of a future policy switch whose timing or composition is uncertain will affect economic dynamics before the switch takes place.

Because of the importance of the budget deficit as a possible cause of inflation, we pay particular attention to the relationship between these two variables. A careful analysis of this relationship is important, because the failure to observe a strong positive correlation between budget deficits and inflation led to the argument that budget deficits are not the cause of inflation (see, for example, Liviatan and Piterman (1986)). Consequently, it was argued that in such cases a disinflation programme need not rely on budget balancing. The Brazilian Cruzado plan of 1986, which failed, is a prominent example of this approach. We will show that even when the budget deficit is the clear source of inflationary pressures, economic theory predicts no simple correlation between the rate of inflation and the size of the budget deficit. Reducing inflation requires a reduction of the budget deficit in such cases, the lack of positive correlation notwithstanding.

We analyse these questions by considering a model where for the existing level of government debt, the government's choice of expenditures, a rate of monetary expansion and level of taxation are inconsistent with ever attaining a steady state. Specifically, the level of seigniorage and regular tax revenues are too low to finance government expenditures and debt service by themselves. For unchanged macroeconomic policies, government debt will therefore grow faster than the interest rate, implying an unsustainable path.

Our primary interest is in the case where the timing of a policy switch is uncertain. The case of certainty about the date of a policy switch (as well as about the policy mix)
is a useful point of comparison and may be derived formally as a special case of timing uncertainty. We study only a closed economy. Open economy issues, and in particular the role of exchange rate management, are discussed in Drazen and Helpman (1987, 1988).

The main results are as follows. Under full certainty, stabilization via an increase in the rate of monetary growth will imply a monotonically increasing inflation rate (and a monotonically declining level of real balances) until the policy switch occurs, while stabilization via increased non-distortionary taxes will yield a constant level of real balances and inflation both before and after the switch. Stabilization via public spending cuts induces a jump in the price level and the real interest factor at the date of stabilization, which reflects the upward jump in private consumption consistent with market clearing. (The nominal interest factor, however, does not jump.) Real balances may rise, stay constant, or fall before a stabilization, with inflation moving in the opposite direction. Hence, the correlation of inflation and the budget deficit depends on the public’s expectation about the policies that will effect a stabilization.

If the switch date is known, but the post-switch policy mix is not, real balances and inflation before the regime switch will follow a path in between the cases above. The exact nature of the path will depend on the subjective probability assigned to various combinations of policies used to effect the stabilization. There will in general be a one-time price jump (which may be either up or down) at the time of the regime switch.

When there is uncertainty about the timing of the switch, the inflation rate will most likely exhibit fluctuations and may overshoot its steady state value, even when real balances move monotonically. Therefore uncertainty about the timing of stabilization can of itself induce fluctuations in the inflation rate. There exist beliefs about the probability of a policy switch which induce very rapid changes in the rate of inflation, and there will almost surely be price jumps following a stabilization. When public spending cuts whose timing is uncertain are used to stabilize, real balances may rise or fall monotonically, or may oscillate, with the rate of inflation similarly exhibiting a wide range of paths. The results on uncertainty strengthen our finding that there need be no positive correlation between budget deficits and the rate of inflation.

The basic structure is set out in the next section. Sections 3, 4, and 5 consider stabilization via monetary, fiscal, and mixed policies respectively. The final section contains concluding comments.

2. BASIC STRUCTURE

In this section we set out the basic model for the case of an uncertain switch date. Individual maximization combined with government behaviour will yield the dynamic equations as well as the boundary conditions which must be satisfied at the time of a policy switch. The regime switch at time $T$ can be described as follows. Prior to $T$ the rate of money growth $\mu$, lump-sum taxes $\tau$, and government spending $g$ are constant at levels which imply increasing government indebtedness. The level of per capita bond holding at $T$, $b_T = b(T)$, will be frozen by choice of a new monetary growth rate $\mu_T$, a new tax rate $\tau_T$, or a new level of government spending $g_T$, which allow government spending plus debt service to be financed by a constant level of taxes (inflation and regular) with no further growth in debt.

A. Individual maximization

The individual is assumed to derive utility from consumption and real money balances, where his instantaneous utility function is assumed separable across commodities and
across time. Suppose a policy switch takes place at time $T$. Since all real variables will be constant from that point onward, we may write the discounted flow of utility as

$$
\int_0^T e^{-\beta t} \left[ u(c(t)) + v \left( \frac{M(t)}{P(t)} \right) \right] dt + e^{-\beta T} V_s \left[ b(T) + \frac{M(T)}{P_s(T)} ; T \right],
$$

where $c$, $M$, and $P$ are real consumption, nominal balances, and the price level, and $\beta$ is the subjective discount rate. The functions $u(\cdot)$ and $v(\cdot)$ are assumed to be increasing and concave. $V_s[\cdot]$ is the discounted flow of utility as of time $T$ from $T$ to infinity (which depends on wealth at time $T$ and possibly the time itself). A subscript $s$ on a function or a variable denotes its value at time $T$ contingent on stabilization taking place at this particular point in time, while unsubscripted variables and functions denote values that obtain at a particular point in time if no stabilization has taken place before or at this point in time. Thus, the price level at time $T$ is $P_s(T)$ if stabilization takes place at time $T$, and $P(T)$ in the absence of a policy switch before or at time $T$. This convention will be used in what follows. Observe that $b(T)$ and $M(T)$ appear in the function $V_s[\cdot]$, which reflects the fact that in the policy experiments that we consider there can be no discrete portfolio shift, so that $b_s(T) = b(T)$ and $M_s(T) = M(T)$. (We consider such shifts in Helpman and Drazen (1988).)

The public forms a probability distribution over possible stabilization dates and acts on the basis of these beliefs. Though in general there is uncertainty about both the timing and the composition of a stabilization package, we assume until Section 5 that the latter is known, in order to isolate the effects of timing uncertainty. In fact, we will concentrate on the case in which a single instrument is used.

To derive the probability distribution, note that there is a maximum level of revenues that can be collected from each tax instrument (because there are limited resources); similarly, there is a minimum feasible level of government expenditures (i.e. zero). There will therefore be a maximum level of debt service, and hence debt, consistent with equilibrium. With debt growing monotonically before a stabilization, this implies an upper bound on the time at which stabilization can take place. We denote by $T_{\text{max}}$ the latest time at which a policy switch is expected, with this time possibly being before the last feasible date of stabilization. Let $F(T)$ be the public's probability distribution function over the timing of stabilization, defined over the time interval $[0, T_{\text{max}}]$. That is, $F(T)$ represents the probability that the policy switch will take place before or at time $T$. Then clearly $F(0) = 0$ and $F(T_{\text{max}}) = 1$.

On the basis of (1), the individual's objective under timing uncertainty which is the expected discounted flow of utility—can be written as:

$$
\int_0^{T_{\text{max}}} \left[ \int_0^T e^{-\beta t} \left[ u(c(t)) + v \left( \frac{M(t)}{P(t)} \right) \right] dt + e^{-\beta T} V_s \left[ b(T) + \frac{M(T)}{P_s(T)} ; T \right] \right] dF(T).
$$

We assume that total output $y_0$ is exogenously given, so that a fixed level of lump-sum taxes implies that after-tax income is fixed as well. The individual can hold either money balances or real bonds $b(t)$ as assets, the real interest rate on the latter being $r(t)$. There are no other assets.

If we define by $z(t)$ the addition to nominal cash balances at time $t$, the individual's choice problem may be thought of as choosing functions $c(t)$, $M(t)$, and $z(t)$ to maximize (2) subject to two constraints, one on income, the other on the relation of $M$ and $z$.

The income constraint is that the present discounted value of income between 0 and $t$ plus the value of wealth at 0 must equal the sum of initial real money balances, the value of bonds at $t$ discounted to time zero, and the discounted value of expenditures
on $c$ and $z/P$. (Bonds are used to bridge the gap between income and expenditure.) This must hold for all $t$. We therefore obtain

$$\frac{M(0)}{P(0)} + e^{-R(t)b(t)} + \int_0^t e^{-R(x)} \left[ c(x) + \frac{z(x)}{P(x)} + \tau(x) - y_0 \right] dx = w(0),$$

where wealth $w(0)$ equals initial bonds plus the initial value of nominal balances evaluated at the initial price level, and where $R(t)$ is the interest factor from time 0 to $t$. The second constraint is

$$M(t) = M(0) + \int_0^t z(x) dx.$$

These constraints should be interpreted as applying up to time $t$ as long as no stabilization has taken place prior to $t$.

The specification of these constraints does not allow for discrete portfolio reallocations (except possibly at time 0), such as an instantaneous shift from money to bonds. This limitation does not affect the results because no shifts of this nature will in fact take place in equilibrium. We discuss such possibilities in Helpman and Drazen (1988), where we consider open market operations. We do however need to add a solvency requirement, namely, $\lim_{t \to \infty} e^{-R(t)b(t)} = 0$.

Maximization of (2) subject to (3) and (4) and the solvency requirement, will yield a solution for the choice variables at each point in time, with the solution being applicable as long as no policy switch has taken place. Using the fact that the marginal utility of wealth $V_t[\cdot]$ is equal to the post-stabilization marginal utility of consumption, the first-order conditions for this problem imply (see the Appendix):

$$e^{R(t)-\beta t} \theta(t) = \int_t^{T_{\text{max}}} e^{R(T)-\beta T} \theta_s(T) \frac{dF(T)}{1-F(t)}$$

$$\frac{1}{P(t)} \frac{1}{\theta(t)} \int_t^{T_{\text{max}}} \left[ \int_t^T e^{-\beta(x-t)} V'[m(x)] \frac{1}{P(x)} dx + e^{-\beta(T-t)} \theta_s(T) \frac{dF(T)}{1-F(t)} \right]$$

where the marginal utility of consumption at $t$, $\theta(t)$, equals $u'(y_0 - g(t))$, since market clearing requires $c + g = y_0$.

In what follows we assume that the distribution function $F(t)$ is differentiable for all $t < T_{\text{max}}$; discontinuous functions are discussed in our working paper. Our assumption does not exclude the possibility of a mass point at $T_{\text{max}}$. Observe, however, that equations (5) and (6) are valid for all types of distribution functions. Under this assumption differentiation of (5) yields:

$$R' = \beta + \frac{\theta - \theta_s}{\theta} \frac{F'}{1-F} \quad \text{for } t < T_{\text{max}}.$$

The left-hand side represents the real interest rate as long as no policy switch has taken place. Hence, the real interest rate is equal to the discount rate plus a risk premium. The risk premium is the product of the instantaneous probability of a regime switch at $t$ conditional on one not having previously taken place, $F'(1-F)$ (that is, the hazard rate), and the percentage fall in marginal utility of consumption which the regime switch induces, $(\theta - \theta_s)/\theta$. If, for example, both of these terms are rising over time, the real interest rate will be rising as well. After the regime switch the real interest rate is equal to the subjective discount rate.
In the case of a (non-distortionary) tax-based or money-based stabilization, the marginal utility of consumption is the same before and after a stabilization, so that the risk premium is zero and the real interest rate is constant and equal to $\beta$; stabilization does not involve a jump in the real interest rate. In the case of an expected budget cut the real interest rate includes a risk premium reflecting the probability of a jump in the marginal utility of consumption. The post-stabilization marginal utility of consumption will be smaller the later stabilization takes place, since the longer that no stabilization has taken place the lower must be government post-stabilization expenditures in order to finance the larger debt service, and hence the higher must be private consumption. Under certainty, (5) implies that a jump in the marginal utility of consumption will yield a jump in the interest factor. We return to the importance of this below.

Equation (6) represents an asset pricing equation for money balances; the real value of a unit of currency equals the expected present value of dividends in the form of marginal utility of real balance holdings plus its resale value. To understand the second component, observe that the utility value as of time $t$ of one unit of consumption at time $T$ when the policy switch takes place at that time is $\exp \left[ -\beta(T-t) \right] \theta(T)$. Since one unit of currency buys $1/P_s(T)$ units of goods if a policy switch takes place at time $T$, the resale value of a unit of currency at time $T$ when a policy switch takes place at this time is $\exp \left[ -\beta(T-t) \right] \theta(T)/P_s(T)$. The expected value is calculated using the conditional probability distribution, that is, conditional on no switch taking place prior to $t$.

Differentiating (6) with respect to $t$, and making use of (7), implies:

$$ v'(m) \frac{\theta}{\theta} = \beta + \frac{F'}{1-F} \left( 1 - \frac{\theta}{\theta} \right) + \frac{\dot{P}}{P} \frac{F'}{1-F} \left( 1 - \frac{P}{P_s} \right) \frac{\theta}{\theta} \quad \text{for} \quad t < T_{\text{max}}. \quad (8) $$

The right-hand side is the nominal interest rate, which includes both a nominal and real risk premium. The second right-hand side term is the real risk premium associated with jumps in the real interest rate that was discussed above. The last term is a nominal risk premium reflecting changes in the real value of money due to a regime switch. It includes three effects: the hazard rate, the percentage change in the real value of nominal balances from a price jump $((1/P - 1/P_s)/1/P)$, and the change in the utility value of real balances $(\theta/\theta)$. Under certainty, when all the probability mass is concentrated on one point, the right-hand side of (8) is simply $\beta + (\dot{P}/P)$.

Since $\theta(t) = u'(y_0 - g) = \theta$ for $t < T$ and $\theta(t) = u'(y_0 - g_T) = \theta_T$ for $t \geq T$, a further implication of (6) is that a cut in government expenditure at time $T$, even if perfectly foreseen, will induce a jump in the price level. This may be seen by noting that (5) and (6) imply

$$ \frac{P(T^-)}{P(T)} = \frac{\theta}{\theta_T} = \frac{u'(y_0 - g)}{u'(y_0 - g_T)}, \quad (9) $$

so that $g_T < g$ implies a (fully anticipated) downward jump in the price level due to the upward jump in consumption necessary to clear the output market.

To help explain the price level jump under perfect foresight, consider a continuous-time Arrow-Debreu economy in which identical individuals have additively separable preferences over time and identical endowments of a single good in each period. The Arrow-Debreu prices as of time 0 of goods to be delivered at future points in time decline at a rate equal to the subjective discount rate. The price function is continuous, with the discount factor equal to the log of the price.

Now suppose instead that the endowment is constant up to a point in time, and constant thereafter at a higher level. Then the price function will itself be discontinuous
at the point where the endowment changes. Consequently, the log of the price, which is the discount factor, is also discontinuous at the point at which the endowment jumps. At this point the real interest factor jumps, and the real interest rate is not defined. One may note that classical methods of pointwise optimization, such as we use in the Appendix, are consistent with this sort of discontinuity.

The next thing to observe is that the nominal interest factor is continuous, even when the real interest factor is not. This stems from the fact that the nominal interest rate equals \( v'(m) / \theta \) at each point in time. In the absence of timing uncertainty this equals in turn to \( \beta + r \) plus the rate of inflation (see (8) and (9), which ensure that this relationship also holds at the switch date). Hence, the nominal interest rate may jump, but the nominal interest factor cannot. This result can be verified directly by solving the problem with a nominal budget constraint in which the nominal interest factor appears explicitly.

The jump in the price level is robust to a number of reasonable alternative specifications of the output market. First, suppose the economy were open, so that goods could be imported from abroad at fixed terms of trade. As long as some part of the cut in government expenditures fell on nontraded goods, there would still be a jump in the price of nontraded goods and a jump in the interest factor in terms of nontradables. The overall price level would therefore also jump (see Drazen and Helpman (1987)).

A second modification would be to allow investment—so that a fixed level of output does not automatically imply a one-to-one relation between changes in private and government consumption. However, in this case too there will be a jump in consumption and in the marginal utility of consumption at \( T \) as long as there are convex adjustment costs to investment.

It is reasonable to suppose that the hazard rate \( F'(1-F) \), representing the instantaneous density of a regime switch conditional on no switch having taken place so far, depends on the economic conditions at each point in time. In particular, it may depend on the size of outstanding government debt, with larger debt levels making a regime switch more likely. We therefore assume that the hazard rate depends on outstanding debt, and, for simplicity, also assume that it depends on no other variables. Hence,

\[
\frac{F'(t)}{1-F(t)} = \phi(b(t)) \quad \text{for } 0 \leq t < T_{\text{max}},
\]

where the above arguments imply that \( \phi'(b) \geq 0 \). The restriction that \( F(T_{\text{max}}) = 1 \) implies that \( \phi(\cdot) \) approaches infinity as \( b \) approaches the upper limit on debt consistent with stabilization—which we denote by \( b_{\text{max}} (= b(T_{\text{max}})) \)—unless there is a mass point at \( T_{\text{max}} \).

**B. Government behaviour and the dynamic equations**

The government's budget constraint may be written

\[
\int_{t}^{\infty} e^{-[(R(x)-R(t)]} [\mu(x)m(x) + \tau(x) - g(x)] dx = b(t).
\]

1. This may be shown by solving the differential equation (10) to yield

\[
F(t) = 1 - \exp \left[ - \int_{0}^{T_{\text{max}}} \phi(b(x))dx \right],
\]

where we have used the boundary condition \( F(0) = 0 \). The other boundary condition, \( F(T_{\text{max}}) = 1 \), implies that \( \int_{0}^{T_{\text{max}}} \phi(b(x))dx = +\infty \), which requires \( \phi \to +\infty \) as \( b \to b_{\text{max}} \), unless there is mass at \( T_{\text{max}} \).
The left-hand side is the present value of the excess of income over spending, where income consists of revenues from the inflation tax \( \mu m \) and lump-sum taxes \( \tau \), while the right-hand side is outstanding debt.

The government controls the rate of money growth, the level of lump-sum taxes, and the level of public spending. In our experiments the rate of money growth, lump-sum taxes, and the level of government spending are constant on the time interval \([0, T]\) at the level \((\mu, \tau, g)\) and at the level \((\mu_T, \tau_T, g_T)\) on the time interval \([T, +\infty)\), where the latter triple is such as to imply that \( b(t) = b_T \) for \( t \geq T \). Hence, all real variables are constant on the time interval \([T, +\infty)\), and so is the rate of inflation. Using (11), the post-stabilization version of (7) and (8), steady state values, which apply after the policy switch, are given by

\[
\frac{v'(m_T)}{\theta_T} = \beta + \mu_T \quad \text{for} \quad t \geq T, \tag{12}
\]

\[
\beta b_T = \mu_T m_T + \tau_T - g_T \quad \text{for} \quad t \geq T. \tag{13}
\]

Given the instrument that will be used to put the economy in a steady state, the timing of the change will imply a particular debt level \( b_T \). The larger is \( T \), the larger this level of debt under the assumptions that we will use. Hence, there exists a one-to-one association between the timing of stabilization and the debt level at which the stabilization will take place (for a given instrument). There is therefore no loss of generality in discussing stabilization at a particular debt level \( b_T \) instead of a particular time \( T \). The former is more convenient for our diagrammatic exposition and is used in what follows.

We can now derive the dynamic equations for debt and real balances prior to stabilization. In order to write them in autonomous form, we note first that the post-stabilization real balance holdings and marginal utility of consumption can be expressed as functions of the debt level. This follows from (12) and (13) together with the definition \( \theta_T = u'(y_0 - g_T) \), implying functional relationships between the marginal utility of consumption and debt and real balance holdings and debt, given the instrument that will be used for stabilization. We denote these relations by \( \theta_s(b_s) \) and \( m_s(b_s) \). Since these functions depend on the instrument that is used for stabilization, we will add in due course a superscript in order to indicate the relevant instrument to which these functions apply. We do not use the superscript as long as the analysis does not involve a particular policy experiment. Also, since there is no jump in debt at the instant of stabilization, we drop the subscript \( s \) from the argument of these functions. Thus, for example, \( \theta^* (b) \) represents the post-stabilization marginal utility of consumption if a tax-based policy switch takes place when the debt level is \( b \).

First, we differentiate (11) with respect to time, making use of (7), (10) and the above defined functions, to obtain:

\[
b = \left[ \beta + \phi(b) \left( 1 - \frac{\theta_s(b)}{\theta} \right) \right] b + g - \tau - \mu m \quad \text{for} \quad t < T_{\text{max}}. \tag{14}
\]

In the case of certainty, the hazard rate is zero before a stabilization, so the term in brackets reduces to \( \beta \). Uncertainty about timing thus implies that the real interest rate depends on the debt level, since debt affects the real risk premium through its effect on both the instantaneous conditional probability of a policy switch and the post stabilization marginal utility of consumption. (In the current framework the latter is relevant only in the case of a budget cut.) Next, combining (8) with the relationship \( \dot{m} / m = \mu - \dot{P} / P \), making use of (9) and the post-stabilization marginal utility and real balance functions,
and taking account of there being no jump in nominal money balances in response to stabilization, we obtain:

\[
\frac{\dot{m}}{m} = \beta + \mu - \frac{v'(m)}{\theta} + \phi(b) \left[ 1 - \frac{m_s(b)}{m} \frac{\theta_s(b)}{\theta} \right] \quad \text{for } t < T_{\text{max}}.
\]

(15)

Under certainty, the last term drops out due to a zero hazard rate before \( T \).

Equations (14) and (15), which apply as long as no stabilization has taken place, form an autonomous system in \( m \) and \( b \) which is used in what follows to derive equilibrium time paths. One may easily show that the stationary state consistent with these equations is a point of unstable equilibrium, that is, a source. For any values of \( \mu, \tau, \) and \( g \) there is only one value of debt which is consistent with steady state. If, for given \( (\mu, \tau, g) \) the values of debt and real balances are such that we are to the right of the \( b = 0 \) locus, debt will grow without bond in the absence of a regime switch. We only consider initial values of debt which are to the right of the steady state point described in Figure 1, that is, \( b_0 > b \).

![Figure 1](image)

3. STABILIZATION VIA MONETARY POLICY

In the case of a stabilization effected by a change in the rate of monetary growth, the government chooses \( \mu_T \) so as to ensure no further growth of debt, taxes and expenditures remaining unchanged. The absence of a change in government spending implies that the marginal utility of consumption is also unchanged, that is, \( \theta''_{\mu}(b) = 0 \). Under these circumstances one can see from (12) and (13) how the post-stabilization rate of money growth determines the sustainable debt level and real balance holdings after \( T \). An
increase in the rate of money growth leads to a decline in real balance holdings, while
the sustainable debt level increases if and only if the elasticity of money demand with
respect to the post-stabilization inflation rate $\mu_T$ (which is implicit in (12)) is smaller
than one.

Curve $m_\nu^*(b)$ in Figure 1 describes the values of debt and real money balances that
can be sustained in a steady state by means of constant rates of money growth. Each
point on this curve is associated with a particular rate of money growth. The solid part
of the curve represents points at which the inflation elasticity of money demand is smaller
than one while the broken part represents points at which this elasticity is larger than
one. The largest debt level sustainable by money financing is $\overline{b}$, which requires the use
of the maximum inflation tax. As is well known this is achieved with the rate of money
growth that implies a unitary elasticity of money demand with respect to the inflation rate.

We assume that the government uses the lowest rate of money growth required to
sustain a given debt level. In this case only the solid part of the terminal surface $m_\nu^*(b)$
is relevant. Hence, if for example the stabilization date $T$ is known with certainty, then
there exists a terminal debt level $b_T$ and there is precisely one point on the terminal
surface; i.e. $[b_T, m_\nu^*(b_T)]$, at which the economy has to be at time $T$. It cannot jump to
this point because under these circumstances there can be no discrete changes in debt or
real balances. The former is obvious, while the latter stems from the fact that the
government makes no discrete changes in the quantity of money, and the equilibrium
price level cannot jump as long as the marginal utility of consumption does not jump.
Therefore this point has to be approached smoothly, which determines uniquely the
equilibrium trajectory (see Helpman and Drazen 1988). We show below that in the
presence of timing uncertainty there can be jumps to the terminal surface.

Constant marginal utility of consumption implies that the real interest rate in the
differential equation of debt, (14), is $\beta$. Equation (15) becomes

$$\frac{\dot{m}}{m} = \beta + \mu - \frac{v'(m)}{\theta} + \phi(b)\left[1 - \frac{m_\nu^*(b)}{m}\right],$$

where $m_\nu^*(\cdot)$ is a decreasing function as long as the government chooses the lowest rate
of monetary growth that is consistent with stabilization. If we assume that the hazard
rate is zero for debt levels lower or equal to the steady state debt level $\underline{b}$, (16) implies
that for $b \leq \underline{b}$ the locus of points $\dot{m} = 0$ is horizontal. For higher values of debt this locus
is downward sloping and located above the $m_\nu^*(b)$ curve, as shown in Figure 1.\(^2\)

For the certainty case, the dynamics are relatively simple. The approach is by means
of a unique path similar to the downward-sloping arrow path in Figure 1, with the path
converging on $m_\nu^*(b)$ at the debt level $b_T$. One may easily show that expectations of a
money-financed stabilization lead to a time pattern of rising debt, declining real balances,
rising nominal interest rate, and rising inflation rate prior to $T$. (The reader can work
out these details, which may also be found in Drazen (1985) or Helpman and Drazen
(1988).) Rising debt leads to a rising budget deficit (inclusive of debt service), and
therefore to a positive association between budget deficits and inflation.

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2. Proof. At point A in Figure 1 the sum of the first three right hand side terms in (16) is zero and the
fourth term is also zero. Now suppose that at some debt level larger than $b$ the locus $\dot{m} = 0$ is below the terminal
surface $m_\nu^*(b)$. Since $v'(\cdot)$ and $m_\nu^*(\cdot)$ are declining functions, the sum of the first three terms on the right
hand side of (16) is negative at this point on $\dot{m} = 0$ and the fourth is negative as well, implying that the right
hand side is negative. This contradicts $\dot{m} = 0$. Therefore the $\dot{m} = 0$ locus has to be above the terminal surface.
However, under these circumstances the right hand side of (16) is increasing in debt. Since it is also increasing
in real balances, the locus $\dot{m} = 0$ is downward sloping.
For the case of timing uncertainty, we may specify the equilibrium dynamic path by working backwards. At the moment of stabilization the system jumps to the terminal surface if stabilization was not anticipated with probability one immediately beforehand. If it was so anticipated, the terminal surface must be reached smoothly. This is as in the certainty case. Consider the time $T_{max}$. If $T_{max}$ has been reached and no policy switch has occurred, then it is known with probability one that a policy switch must take place at $T_{max}$. Therefore, point $B$ in Figure 1 must be approached smoothly. This is seen formally from (6) by observing that as $t$ approaches $T_{max}$ the price level $P(t)$ approaches $P_{r}(T_{max})$. Hence, stabilization cannot bring about a price jump if it takes place at the last possible moment, and for this reason real balances also cannot jump. This establishes that point $B$ is on the equilibrium trajectory. Working backwards by means of (16) and the relevant version of (14), we obtain the downward-sloping arrow path as the equilibrium trajectory of debt and real balances. This trajectory applies as long as no policy switch has taken place.

If no policy switch takes place until point $B$ is reached, stabilization freezes the economy at $B$. If, on the other hand, stabilization takes place when point $C$ is reached, it freezes the economy at point $D$ on the terminal surface. The jump from $C$ to $D$ is effected by an unexpected upward price jump (implying a downward jump in real balances). The direction of the price jump obviously depends on whether the dynamic path for real balances is above or below the terminal surface. It can be shown that the dynamic path is below the terminal surface for sufficiently high debt levels if $b_{max}$ is sufficiently close to $\bar{b}$, the debt level that can be sustained by the maximum inflation tax, as in Figure 1. (See Drazen and Helpman (1986).) The trajectory in the figure describes a situation in which a regime switch "late in the game" induces a downward price jump while a regime switch fairly early on induces an upward jump.

Monotonically declining real balances until the switch takes place imply a rising nominal interest rate, which in the case where the date of the switch is known with certainty implies a monotonically rising inflation rate. This result does not carry over to the uncertainty case: a rising nominal interest rate prior to stabilization does not imply a rising inflation rate as well. The reason is that the risk premium might be rising so fast as to bring about a declining rate of inflation. This possibility is seen from writing the equilibrium rate of inflation (derived from (16)) as

$$\frac{\dot{P}}{P} = -\beta + \frac{v[m(t)]}{\theta} - \phi[b(t)] \left[ 1 - \frac{m_{\mu} [b(t)]}{m(t)} \right].$$

(17)

Hence, falling real balances and a rising risk premium may be associated with a declining rate of inflation.

We also note that a large negative risk premium may imply a large positive rate of inflation. Therefore, during the pre-switch period the rate of inflation may be higher than in the post-switch equilibrium. Moreover, cyclical movements of the risk premium may induce cyclical movements in the rate of inflation. In the determination of the inflation rate the risk premium becomes especially important as time $T_{max}$ is approached if the hazard rate is approaching infinity there (which is necessarily the case when the distribution function has no mass at $T_{max}$). Then instantaneous changes in the risk premium become the dominant factor in determining movements in the rate of inflation and may well induce "overshooting."

To examine these presumptions, we ran numerous simulations of the model. For a range of values for the exogenous variables, we indeed found there to be overshooting in the rate of inflation as debt approached its upper limit. The results of one simulation
may be seen in Table 1 (this is a sample from several thousand points) which presents
the relationship between debt, real balances, the rate of inflation and the risk premium.
The relationship between debt and the rate of inflation is plotted in Figure 2, showing a
rise in the rate of inflation followed by a rapid fall as debt approaches its upper limit.
This makes clear that in the presence of timing uncertainty a monetary stabilizations does
not necessarily lead to a positive comovement of the budget deficit and the rate of inflation,
as is evident from Figure 2 as time approaches $T_{\text{max}}$.

Our analysis bears on Sargent and Wallace's (1981) "Unpleasant Monetarist Arithmetic". They consider an economy initially in steady state with a constant rate of inflation and money growth in which the government unexpectedly reduces the rate of money growth without changing taxes or expenditure. This induces a growth in debt which is stopped only when the rate of monetary growth is raised to the necessary level. They ask what are the inflationary consequences of a current tightening of monetary growth, given the anticipation of an eventual inevitable increase in the money growth rate. As Drazen (1985) has shown, the temporary tightening of money growth can bring about an initial increase or reduction in the rate of inflation, but thereafter the rate of inflation will rise over time until it reaches its new higher steady state level. There may, but need not be, short-run decreases in the rate of inflation. It is in fact possible for the tight money policy to bring about higher inflation at all times.

The Sargent-Wallace analysis assumes that the date at which debt is stopped from
growing is known with certainty. Our analysis thus implies that in the presence of
uncertainty about this date, a tight money policy can generate temporarily very high
inflation rates, with the rate of inflation declining over time as long as debt continues to
grow (see the last part of the graph in Figure 2). This is more likely the larger are the

### Table 1

*An example of overshooting*

$v'(m) = \begin{cases} 
-\ln \left( \frac{m}{100} \right), & 0 \leq m \leq 1, \\
0, & m \geq 1,
\end{cases}
\phi(b) = 1 + 100(\bar{b} - b)^{-2},

$b = 24 - 266, \quad b_{\text{max}} = 23 - 788.$

<table>
<thead>
<tr>
<th>$b$</th>
<th>$m$</th>
<th>$\frac{\dot{p}}{p}$</th>
<th>$\frac{\phi \left( 1 - \frac{m^\mu}{m} \right)}{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.885</td>
<td>30.415</td>
<td>0.8018</td>
<td>-0.1116</td>
</tr>
<tr>
<td>22.002</td>
<td>30.227</td>
<td>0.8170</td>
<td>-0.1206</td>
</tr>
<tr>
<td>22.244</td>
<td>29.983</td>
<td>0.8516</td>
<td>-0.1418</td>
</tr>
<tr>
<td>22.370</td>
<td>29.612</td>
<td>0.8719</td>
<td>-0.1549</td>
</tr>
<tr>
<td>22.500</td>
<td>29.385</td>
<td>0.8946</td>
<td>-0.1699</td>
</tr>
<tr>
<td>22.633</td>
<td>29.147</td>
<td>0.9202</td>
<td>-0.1874</td>
</tr>
<tr>
<td>22.771</td>
<td>28.995</td>
<td>0.9490</td>
<td>-0.2074</td>
</tr>
<tr>
<td>22.912</td>
<td>28.827</td>
<td>0.9823</td>
<td>-0.2315</td>
</tr>
<tr>
<td>23.207</td>
<td>28.054</td>
<td>1.0677</td>
<td>-0.2960</td>
</tr>
<tr>
<td>23.351</td>
<td>27.702</td>
<td>1.1232</td>
<td>-0.3395</td>
</tr>
<tr>
<td>23.362</td>
<td>27.738</td>
<td>1.1794</td>
<td>-0.3840</td>
</tr>
<tr>
<td>23.701</td>
<td>26.928</td>
<td>1.1751</td>
<td>-0.3629</td>
</tr>
<tr>
<td>23.753</td>
<td>26.823</td>
<td>1.0393</td>
<td>-0.2233</td>
</tr>
<tr>
<td>23.788</td>
<td>26.776</td>
<td>0.8177</td>
<td>0</td>
</tr>
</tbody>
</table>
initial debt level and the instantaneous conditional probability of a policy change, that is, the less confidence there is in the tight money policy.  

4. STABILIZATION VIA FISCAL POLICY

It is easy to see from (12) that since lump-sum tax increases do not affect the marginal utility of consumption, they will not affect the level of real balances consistent with steady state. Hence, the terminal surface for tax changes, denoted by \( m_s(b) \), is a horizontal line, coinciding with the certainty version of (15). Higher taxes are associated with points further to the right on this surface.

The dynamic equations are identical to the case of a money-based stabilization, except that \( m_s(b) \) replaces \( \hat{m}(b) \). Since the dynamic path must intersect \( m_s(b) \) at \( T_{\text{max}} \), which is identical to the \( \hat{m} = 0 \) locus under certainty, the path must always be on this locus. Therefore, the dynamic equations and results turn out to be identical in the certainty and uncertainty case. Real balances are constant and debt is growing over time. The former implies a constant nominal interest rate and a constant rate of inflation that equals the rate of money growth. The values of all these variables do not change as a result of stabilization. There is Ricardian equivalence, because taxes are non-distortionary. The

3. If the utility function is log-linear in real balances, the results are identical to the certainty case. This arises because if the dynamic path ever crosses the \( m_s''(b) \) locus, it must remain on this locus. The dynamic path must therefore always be on this locus in the log-linear case. This case is discussed in Drazen and Helpman (1985a). See also Liviatan (1984) and Drazen (1985).
independence of the dynamic trajectory from the timing of stabilization means that uncertainty about timing will not affect the pre-stabilization path.

In Helpman and Drazen (1988) we discussed the case of distortionary labour income taxes in the presence of elastic labour supply and showed that real variables will be affected. In such a case, there will be a negative association between inflation and budget deficits when the interest elasticity of demand for money is smaller than one.

Stabilization via expenditure cuts imply that both real balances and sustainable debt unambiguously rise. (See (12) and (13).) As with stabilization based on changes in money growth or taxes, there is a maximum feasible level of steady state debt which can be supported by a budget cut, as there is a minimum feasible level of expenditures. The resulting terminal surface is described by curve $m^*(b)$ in Figure 3.

The dynamic equations (14) and (15) apply with $m_s(\cdot) = m^*_s(\cdot)$ and with $\theta^*_s(b) = u'[y_0 - g(b)]$, where $g(b)$, which represents steady-state expenditures as a function of debt, is defined as $g(b) = \tau + \mu m^*_s(b) - \beta b$. In this case the $b = 0$ locus is upward sloping, as shown in Figure 3, while the $\dot{m} = 0$ locus may slope either up or down. (Details are in our working paper.) For debt levels larger than $b$ debt is rising to the right of the $b = 0$ locus and falling to the left of it, while real balances are rising above the $\dot{m} = 0$ locus and falling below it.

The key difference between this and earlier cases (except for the case of distortionary labour taxes that was not discussed in detail) is that stabilization via budget cuts implies, even under certainty, a jump in the real interest factor and in the price level, and hence, in real balances. This is due to the jump in the marginal utility of consumption that was discussed in detail in Section 2. Specifically, since there is no discrete change in the quantity of money, under certainty (9) implies

$$\frac{m_T}{m(T^-)} = \frac{\theta}{\theta_T} > 1.$$
Therefore the terminal point is not approached smoothly, but via an upward jump in real balances that results from a downward price jump. This jump may be from a point above, on, or below the \( m = 0 \) locus.

Since the \( m = 0 \) locus is horizontal under certainty, the dynamic path cannot cross it before \( T \), so that real balances must move monotonically, but could rise, stay constant, or fall, in line with the result at the end of the previous paragraph. In the first case, for example, real balances will be rising over time, the inflation rate will be below the rate of money growth and declining over time prior to stabilization, and will jump to the rate of money growth at time \( T \). This corresponds to an interest elasticity of money demand everywhere larger than one, or more generally, depends on \( v'(mT)\) being larger than \( v'(\hat{m})\), where \( \hat{m} \) is the level of real balances at which the line \( m = 0 \) is located.\(^4\)

Under uncertainty, there is a downward price jump if stabilization takes place at \( T_{\text{max}} \), because at \( T_{\text{max}} \) there is no residual uncertainty. The price level jump is proportional to the jump in the marginal utility of consumption. If we define \( m_{\text{max}} = m(T_{\text{max}}) \), then

\[
\frac{\theta}{\theta^*(b_{\text{max}})} = \frac{P(T_{\text{max}})}{P_i(T_{\text{max}})} = \frac{m_{\text{max}}^*(b_{\text{max}})}{m_{\text{max}}}. \tag{19}
\]

This condition determines the endpoint of the dynamic trajectory. The entire trajectory can then be worked out backwards by means of (14) and (15). The key to its behaviour is the location of \( m_{\text{max}} \) relative to the \( m = 0 \) locus. If \( m_{\text{max}} \) lies above it, the dynamic path will be rising as \( t \) approaches \( T_{\text{max}} \); if it lies below, it will be falling. Four possible trajectories are presented in Figure 4. In all cases there is a downward price jump at the last possible moment, because then there is no residual uncertainty and the marginal utility of consumption jumps downward, as in (19). Stabilization prior to that however may bring about an upward or downward price jump.

Since the \( m = 0 \) locus is not horizontal in the uncertainty case, the dynamic path can cross it. If it does cross, real balances will not move monotonically along the path. If it does not, they will. (Debt is monotonically rising along every path.) The former possibility is presented in panels (b) and (d) of Figure 4, the latter in panels (a) and (c).

The behaviour of inflation along any path depends not only on the behaviour of real balances, but also on the risk premia, which reflect the possibility that at any instant a regime change may take place which would induce a jump in both the price level and the real interest rate. For this reason inflation and real balances need not move in opposite directions, just as in the case of a money-based stabilization. Thus, when the public expects an expenditure-cut based stabilization, the rate of inflation may rise, fall, or cycle.

4. Let \( \hat{m} \) be the value of real balances for which \( \hat{m} = 0 \) before stabilization. We have from (12) and (15) under certainty that

\[
\frac{v'(\hat{m})}{\theta} = \beta + \mu = \frac{v'(mT)}{\theta}. \tag{18}
\]

Combining this with (18) and rearranging, we obtain

\[
\frac{m(T^-)}{\hat{m}} = \frac{v'(mT)mT}{v'(\hat{m})\hat{m}}. \tag{19}
\]

It is clear from this that real balances just before the price jump at \( T \) are above the horizontal \( m = 0 \) locus if and only if the right hand side of this equation is larger than one. Now use (8) under certainty to write the demand for real balances in implicit form as \( v'(m) = \theta_i \), where \( i \) is the nominal interest rate. Hence, if the implied elasticity of money demand with respect to the interest rate is everywhere larger than one, then the fact that \( mT > \hat{m} \) implies \( v'(mT)mT > v'(\hat{m})\hat{m} \) and \( m(T^-) > \hat{m} \) (the last inequality relies on the last equation). The other cases can be derived in a similar fashion. These points are treated more fully in Drazen and Helpman (1985b).
prior to the stabilization actually taking place. This strengthens the possibility that inflation and budget deficits will not be positively correlated.

In Drazen and Helpman (1985b) we present an example that replicates the time path shown in panel (b) of Figure 4. This may be of special interest in connection with the European hyperinflations of the 1920's, which were ended by fiscal reforms including sharp cuts in government expenditures, whose timing was uncertain ex-ante. In a number of cases the sharp drop in real balances which characterized hyperinflations was reversed before the fiscal reforms were enacted (Austria and Hungary provide two examples). We think it useful to show that such behaviour can arise in our model, in particular in view of the fact that in that example it was assumed that the instantaneous probability of a switch was constant, so that the time pattern of this variable is not the main driving force behind the result. This lengthy example is not presented in order to save space.

Multistage stabilization programmes—implying more complex time paths—can also be analysed in this framework. The reader is referred to Drazen and Helpman (1985) for some interesting examples.

5. MIXED POLICY STABILIZATIONS

Known combinations of \((\mu_T, \tau_T, g_T)\) to finance a given budget at a known stabilization date \(T\) may be easily analysed. Any policy mix rule consistent with stabilization will
determine a terminal surface relating real balances to debt. When the policy mix is limited to changes in the instruments in the direction of a lower deficit (increase in the rate of money growth and taxes and expenditure cuts), this surface will lie between the terminal loci for budget cuts and money financing, its precise position depending on the policy mix. (Otherwise the terminal locus could lie above \( m^b(b) \) or below \( m^\mu(b) \).) The dynamic path will then be that which just hits this terminal surface at \( T \).

Now suppose that the stabilization date \( T \) is known with certainty, but that there is uncertainty about the policy mix that will be used to stabilize. Our analysis of the certainty case implies that, from the point of view of the individual, the relevant difference between alternative policy mixes is in the post-switch price level \( P_T = P(T) \). Uncertainty about the policy mix induces uncertainty about this price level. Let \( G(P_T) \) be the distribution of the price level at \( T \) that is induced by the subjective probability distribution over possible policy mixes, and knowledge of the structure of the economy linking each policy mix to a value of \( P_T \). The consumer problem is then to maximize expected discounted utility over an infinite horizon, where \( T \) is known and where \( G(P_T) \) is used to form expected utility. Analogous to (2) we may write the objective function as

\[
\int_0^T e^{-\beta t} \left[ u(c(t)) + v \left( \frac{M(t)}{P(t)} \right) \right] dt + e^{-\beta T} \int_{P_T = \infty}^{P_T = 0} V \left[ b(T) + \frac{M(T)}{P_T}; T \right] dG(P_T). \tag{20}
\]

The individual maximizes (20) subject to the same budget constraints (3) and (4). Consequently, the dynamic equations before stabilization are identical to those for the certainty case. Therefore, once the terminal point of the dynamic trajectory is specified, the entire path prior to stabilization will be known.

After the precise policy mix is announced at \( T \), the economy must jump to the terminal surface that corresponds to this policy (that is, given \( b_T \) and the policy mix, \( m_T \) has to satisfy (12) and (13)). The jump of real balances will result from a jump of the

\[ m^* \]

\[ m_{\mu}(b) \]

\[ m^b(b) \]

\[ \hat{b} = 0 \]

\[ \hat{m} = 0 \text{ and } m^*(b) \]

\[ b_0 \]

\[ b \]

\[ b_T \]

\[ \hat{b} \]

\[ 0 \]

\[ b \]

(FIGURE 5)
price level.\textsuperscript{5} The price level before the policy switch will be determined by an asset pricing equation

$$
\frac{1}{P(t)} = \int_t^T e^{-\beta(x-t)} \frac{v'(m(x))}{\theta} \frac{1}{P(x)} \, dx + e^{-\beta(T-t)} \int_0^T \frac{1}{P_T} \, dG(P_T), \quad \text{for } t < T,
$$

that can be derived from the individual’s first-order conditions. Hence, as \( t \) approaches \( T \) the inverse of the price level \( 1/P(t) \) must approach the expected value of \( 1/P_T \). This ties down the price level the instant before \( T \), which, in turn, ties down the whole path. The equilibrium path may then be represented as in Figure 5, with its exact location at \( t = T^- \) dependent on how much weight is assigned to each possible choice of policy mix at \( T \). If, for example, the expectation is that adjustment will come primarily via an increase in the rate of monetary growth, with only small increases in taxes and cuts in expenditures, the path will be relatively closer to the \( m_x^\mu(b) \) curve. If the actual programme then includes a smaller increase in the rate of money growth than was expected, so that the terminal surface is above the optimal path based on the contrary expectations, there will be an upward jump in real balances, or, equivalently, a downward price level jump.

6. SUMMARY AND CONCLUSIONS

The purpose of this paper was to consider situations where current macroeconomic policies are known to be infeasible, implying an eventual regime switch, but where the exact timing or nature of this switch is unknown. Our goal was to consider the effects of this uncertainty on macroeconomic variables.

The results for the specific instruments under both certainty and uncertainty are summarized in the introduction. A central characteristic of the time paths which emerges is the lack of any necessary contemporaneous correlation between budget deficits and the rate of monetary growth on the one hand and the rate of inflation on the other. Depending on the expected policy which effects the regime switch, a rising deficit inclusive of debt service may induce a rising, constant, or falling inflation rate. With uncertainty about the timing of a policy change, constant rate of money growth and a constant deficit exclusive of debt service may be associated with a fluctuating inflation rate. This lack of correlation arises even though the budget deficit is clearly the ultimate cause of inflation.

APPENDIX—DERIVATION OF FIRST-ORDER CONDITIONS

When the cumulative distribution of a switch occurring until \( T \) is \( F(T) \), the individual maximizes (2) in the text, subject to (3) and (4). The choice problem may be written in Lagrangian form as

$$
\max_{t,c(t),z(t),M(t)} \int_0^T \left[ \int_0^T e^{-\beta t} \left[ u(c(t)) + v \left( \frac{M(t)}{P(t)} \right) \right] dt + e^{-\beta T} \left[ e^{R(T)} w(0) - \int_0^T e^{R(T)-R(t)} \left( c(t) + \frac{z(t)}{P(t)} - y \right) dt \right] + \frac{M(0) + \int_0^T z(x) dx}{P(T)} + \int_0^T \gamma(t) \left[ M(0) + \int_0^T z(x) dx - M(t) \right] dt \right] dF(T)
$$

(A.1)

5. The jump in the price level will reflect not only the “arrival of new information” about the actual stabilization programme, but also any change in \( c \) and \( \theta \) if the stabilization includes a budget cut.
where \( \gamma(t) \) is the multiplier of constraint (4) and (3) has been substituted into the \( V_\epsilon(\cdot) \) function. Pointwise maximization of (A.1) with respect to \( c(t), z(t), \) and \( M(t) \) yields, respectively (where \( \theta_\epsilon = u'(y_0 - g) = V_\epsilon \) is the derivative of \( V_\epsilon(\cdot) \) with respect to wealth and \( \theta = u'(y_0 - g) \)),

\[
[1 - F(t)] e^{-\beta t} \theta(t) = \int_{t}^{T_{\max}} e^{-\beta T} e^{R(T) - R(t)} \theta_\epsilon(T) dF(T),
\]

\[
\left[ -\theta_\epsilon(T) e^{-\beta T} e^{R(T) - R(t)} \frac{1}{P(t)} + \theta_\epsilon(T) e^{-\beta T} + \int_{t}^{T} \gamma(x) dx \right] dF(T) = 0,
\]

\[
e^{-\beta t} \frac{1}{P(t)} u'(m(t)) = \gamma(t).
\]

Equation (A.2) implies (5) while the combination of (A.2) through (A.4) implies (6).

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