

A simple macrodynamic monetary model

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1 Introduction

This lecture presents a simple, but fully formed, macrodynamic model, leading to the development of the IS-LM model, and exposes this model to some comparative statics.

Prior to the 1960's, the IS-LM model was the workhorse of macroeconomics¹. Almost every policy decision made prior to about 1965 was made with an IS-LM framework at the forefront of the analysis of the likely effects of the policy in question.

Though flawed, the IS-LM remains a 'back of the envelope' type of model, and is used in practical discussions of policy to this day, so it is worth some time learning where it came from, what it is, and what one can do with it. That's what this lecture is about.

First, though, we should define what we mean by macrodynamics.

Definition 1 (Macrodynamics) *Macrodynamics studies the evolution of the macroeconomy over time. The macroeconomy is assumed to evolve from an initial state towards a steady state, encountering exogenous shocks as it does so. These shocks can be studied using comparative statics and dynamics to come up with policy proposals to help the economy deal with these shocks in the future.*

The dynamics I'm going to show you are pretty simple. In fact, because you'll have seen this model before, it won't seem like I'm doing anything very different at all. The IS-LM model accumulates capital over time, causing its curves to shift in response. This might not seem like a new or terribly interesting idea, but it is, and here's why: when households have a portfolio balance decision to make (how much of it's wealth to hold in cash versus interest bearing bonds), the wrong decision can prove very costly: witness the sub-prime mortgage crisis in the US at the moment, or the resultant credit crunch all over the world. If policy makers can advise households on how to proceed, then everyone can be made better off as a result.

2 The Model

The macroeconomy we study is assumed to be simple enough to divide into four main markets. The variables we want to study are real income, the level of employment, the real interest rate, and the inflation rate. We deal in aggregates, glossing over problems like talking about 'the' rate of interest, when there are in fact millions of them, to produce a tractable model which is capable of yielding some policy insights.

Our model, as I've said, has four parts, or sectors, namely

- The product market,
- The money market,

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¹The classic papers in this area are [Hicks \(1937, 1950\)](#); [Tobin \(1969\)](#) and ([Turnovsky, 1995](#)).

- The bond market,
- The labour market.

Because of Walras' law, we only need to determine equilibria in three of these markets, and the fourth will snap into place. We usually drop consideration of the bond market, and focus on the other three, but this is only a convention: you could solve for any three and have the fourth work out for you.

Starting from the standard national income framework, we take the accounting identity

$$GDP = C + I + G, \tag{1}$$

where C is consumption expenditure by the private sector, I is gross private investment by domestic firms, and G is total government purchases of goods and services. The total produced by these three aggregates in a given period (say, a year), is the gross domestic product of the nation.

In an open economy, equation 1 can be extended to include imports (M) and exports (X):

$$GNP = C + I + G + (X - M). \tag{2}$$

Equation 1 can also be divided into flows of GDP, for example, consider the breakdown of GDP describing how the income earned by the sale of production of goods is disposed of:

$$GDP = C + S + T. \tag{3}$$

Here S is total savings, and T is net tax payments. Equation 3 is telling us that income can either be saved, consumed, or it can be taxed off us.

We can also think about how income is generated by the production process: income can be earned in the form of wages and salaries, or corporate profit, or rental income, dividend income, interest income, and so on. It is important to realise when we are talking about the macroeconomy, everything is *connected*, and, at a sufficiently abstracted level, everything affects everything else, as you'll see when the IS-LM model is solved for it's steady state.

Now, lets describe the three markets in our economy, and solve for their interactions to produce a general macroeconomic equilibrium model.

2.1 The Output Market

Think of this market as the composite of all the 'real' stuff produced in the economy, like hairdryers, football boots, watches, and plastic boxes, as well as haircuts, dramatic productions of Henry V, and bus rides. We can think of this sum of goods and services as one market, and it helps to simplify the algebra if you just think about the economy as producing one good or commodity—say, wheat. In a closed economy, the production or output (Y) of this good is described by

$$Y = C + I + G \tag{4}$$

Where Y is real output, or national income, G is real government expenditure, I is real private demand for investment, and C is real private demand for consumption.

You mustn't confuse equation 4 with equation 1. Equation 1 *has* to hold, because it is an accounting identity. Equation 4 only holds when the economy is in equilibrium. It should be clearly understood that equation 4 is only the product of economy theory—it is what we *think* might happen when all the factors of production are used optimally over a period.

Our next step is to develop what Keynes (1936, rhymes with 'brains') called 'behavioural' relationships between the dependent variable of equation 4, Y , and the independent variables, C , I and G . We can write down a consumption function, which assumes the level of consumption depends on the amount of national income in a given period, holding investment and government spending constant (for which we'll write $I = \bar{I}$ and $G = \bar{G}$). The simplest consumption function, which you will have seen before, looks like this

$$C = C(Y). \quad (5)$$

Substitute 5 into equation 4, and we have

$$Y = C(Y) + \bar{I} + \bar{G}. \quad (6)$$

Now we can solve for Y in terms of exogenously determined values of \bar{I}, \bar{G} and derive a perfectly respectable, if incredibly boring, macroeconomic equilibrium model.

Much research has looked at the appropriate shape the consumption function should take², and we can expand the relationship between income and consumption, and income and investment. Let's say consumption is a function of real disposable income, $YD = Y - T$, so equation 5 becomes

$$C = C(YD). \quad (7)$$

We can go crazy on the formulation of consumption functions in more and more elaborate ways, but let's stop at the formulation adopted by Friedman (1957), and say that consumption should depend on a household's disposable income today, its current wealth (A), and the interest rate r . Plugging these into equation 7, we have

$$C = C(YD, A, r). \quad (8)$$

Now let's turn to the definition of real private wealth, A . In a micro-founded model, we should consider the relationship between the nominal stock of outside money, M , the real stock of physical capital, K , the nominal stock of government bonds, B , and the price of capital goods, P_k . A pretty comprehensive definition is going to look something like equation 9 below:

$$A = \frac{M + P_k B + P_k K}{P}. \quad (9)$$

The level of private investment in the system will be given by another behavioural equation studied to death by the profession, the investment function:

$$I - I(r - \pi). \quad (10)$$

The investment function gives the level of private investment in the economy for a given real interest rate ($r - \pi$), which is the nominal interest rate r minus the expected inflation rate, π . Equation 10 tells us as the interest rate rises, the number of projects a profit-seeking firm will invest in declines, so the relationship between I and $I(r - \pi)$ is negative. Like the consumption function, we can make the formulation of 10 as complicated as we like, but let's stop there. Interested students should read (Turnovsky, 1995, pp. 18–30).

Substituting equations 10 and 9 into our consumption function from equation 5, we have

$$Y = C\left(T - T, r - \pi, \frac{M + B + P_k K}{P}\right) + I(r - \pi) + G. \quad (11)$$

Equation 11 has the price of bonds set to 1, and the number of bonds in the system set to 1, because that is the fourth market we are going to ignore and solve for indirectly via Walras' law. We call equation 11 the IS curve, and it is downward-sloping.

Definition 2 (IS Curve) *The IS curve gives the combinations of Y and r which keep the product market in equilibrium, given the stocks of assets M, B and K .*

²See Friedman (1957) for a survey.

We can see this mathematically by differentiating Y with respect to r .

$$\left(\frac{dr}{dY}\right)_{IS} = \frac{1 - C_1}{I' + C_2}.$$

If we allow $I' + C_2$ to be strictly negative, we have a downwardly sloping curve. As income rises, but by a smaller amount. The only way the excess output can be absorbed is through the additional investment of consumption—and for this to happen, the interest rate has to fall.

The position of the IS curve is dependent on the position of the investment function, the consumption function, and the level of government expenditure.

Look at figure 1 below. The IS curve exists in output/interest rate space, and several things can cause it to move out or in (from IS to I'S', say).

An increase in G , or the consumption or investment functions, will cause the IS curve to shift out. You can shift the curves by

1. an increase the expected rate of inflation, or
2. a fall in the price of output, or
3. a rise in the stock of assets, or
4. an increase in the price of capital, or
5. a reduction in the level of taxes.

2.2 The financial sector

Attaining balance in the goods market alone won't get us to a general equilibrium, so we need to focus on the financial sector. The IS curve, as we've seen, determines pairs of Y and r consistent with conditions in the product market. We still need the financial market to pick out the unique (Y, r) pair.

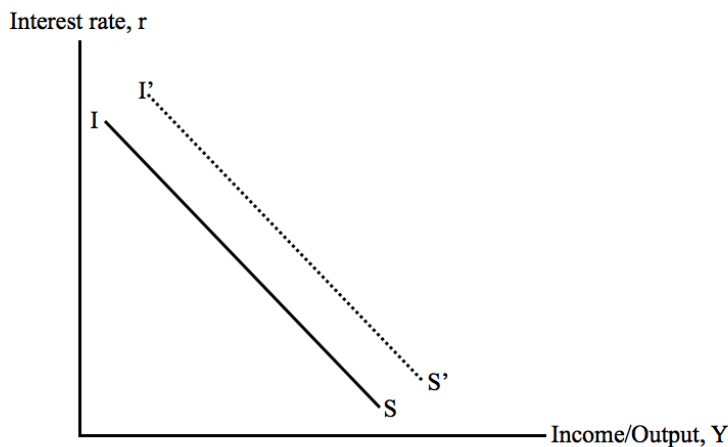


Figure 1: The IS curve.

[Tobin \(1969\)](#) developed portfolio theory to explain household liquidity decisions. His essential insight was individuals in a financial system will act to minimise the risk associated with a level of expected return.

In our macro model, the essential choice comes down to how much the households want to hold in cash, and how much they want to hold in interest bearing securities. We can model this choice using Tobin's original formulation.

Say the economy contains three outside monetary assets: fiat money issued by a government or central bank (M), government bonds (B), and physical capital (K). Tobin's asset system can be specified as

$$\frac{M^D}{P} = L\left(Y, -\pi, r - \pi, r_k, \frac{M + B + P_k K}{P}\right) \quad (12)$$

$$\frac{B^D}{P} = J\left(Y, -\pi, r - \pi, r_k, \frac{M + B + P_k K}{P}\right) \quad (13)$$

$$\frac{P_k K}{P} = N\left(Y, -\pi, r - \pi, r_k, \frac{M + B + P_k K}{P}\right) \quad (14)$$

$$\frac{M^D + B^D + P_k K^D}{P} = \frac{M + B + P_k K}{P} = A \quad (15)$$

$$r_k = \frac{P \times R\left(\frac{Y}{K}\right)}{P_k} \quad (16)$$

Equations 12 13 and 14 give the demand functions for the three assets. The demand functions depend on real output, real rates of return on the three assets, money, bonds, and equities: $-\pi, r - \pi$ and r_k , and real wealth.

Equation 15 is the wealth constraint, which says that if two asset markets are in equilibrium, then the third must be as well. The constraint represented by equation 15 is built from each household's individual constraints, and can be specified in a number of ways which we won't go into here.

Equation 15 relates the real rate of return on a unit of capital r_k to the underlying marginal physical product of capital, R , generating the income stream.

Because by solving for two of the markets we actually solve for all three, we can eliminate one of the assets (say, capital), and substituting in for A and P_k , the financial sector described in the equations above can be reduced to two equations:

$$\frac{M}{P} = L\left[Y, -\pi, r - \pi, r_k, \frac{M + B}{P} + \frac{RK}{r_k}\right] \quad (17)$$

$$\frac{B}{P} J\left[Y, -\pi, r - \pi, r_k, \frac{M + B}{P} + \frac{RK}{r_k}\right] \quad (18)$$

Tobin treats Y, M, P, B and K as given, in which case equations 18 and 17 determine the rate of return on the two remaining assets.

Now for the grand reduction, part one.

Suppose that bonds and capital are perfect substitutes., in which the real rates of return on both assets are the same, so

$$r_k = r - \pi. \quad (19)$$

Equation 19 says our independent asset functions will cease to exist: we'll just have a composite demand function made up of $J(\cdot) + L(\cdot)$ for the single bonds-plus-capital market. The stock constraint given by equation 15 then implies only one independent asset market, which we'll call the money market.

Part two. Assume the asset demand depend on differential rates of return, then $(r - \pi) - \pi = r$, and the money market equilibrium conditions reduce to

$$\frac{M}{P} = L(Y, r, A) \quad (20)$$

If we treat the money supply M , as exogenous, equation 20 is the LM curve.

Definition 3 (LM Curve) *The LM curve determines, for a given price level, P , the combinations of (Y, r) that will keep the money market in equilibrium.*

The LM curve is upward sloping. Differentiating equation 20 with respect to Y gives us

$$\left(\frac{dr}{dY}\right)_{LM} = -\frac{L_1}{L_2} > 0. \tag{21}$$

Figure 2 represents the LM curve. Typically we draw the LM curve in a convex shape, implying that at high rates of interest, speculative balances will be reduced to a minimum, making the market much harder to beat, because at that point any increase in the price level will be accompanied by a relatively large increase in the rate of interest, to free the necessary money for additional transactions. The opposite occurs at very low levels of interest.

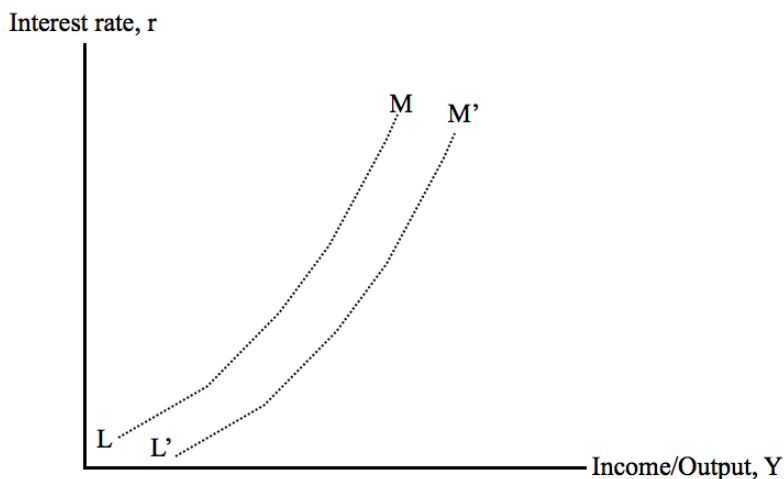


Figure 2: The LM Curve.

2.3 Equilibrium in the product and money markets

Figure 3 gives the equilibrium condition. For given levels of predetermined and exogenous variables, the equilibrium level of income and interest rate is determined by combining the IS and LM curves. We can solve uniquely for (Y, r) in terms of P, G, M, B, T, K, π by slotting together equations 20 and 11 to get a solution equation of

$$Y = Y(P; M, B, K, \pi, G, T). \tag{22}$$

Equation 22 relates national income in equilibrium to demand, as determined by price, and a host of other macroeconomic variables. We can use the IS-LM model to analyse policy changes, by considering the model first in equilibrium, then asking what effects a change in taxes, monetary policy, or government expenditure might have on the model. An excellent source to consult on the movements of the IS LM is Blanchard (2005, pp. 89–109). I’ll also show you the movements of the ISLM in a *Mathematica* simulation in class³.

³The Mathematica simulation can be downloaded from www.wolfram.com/demonstrations.

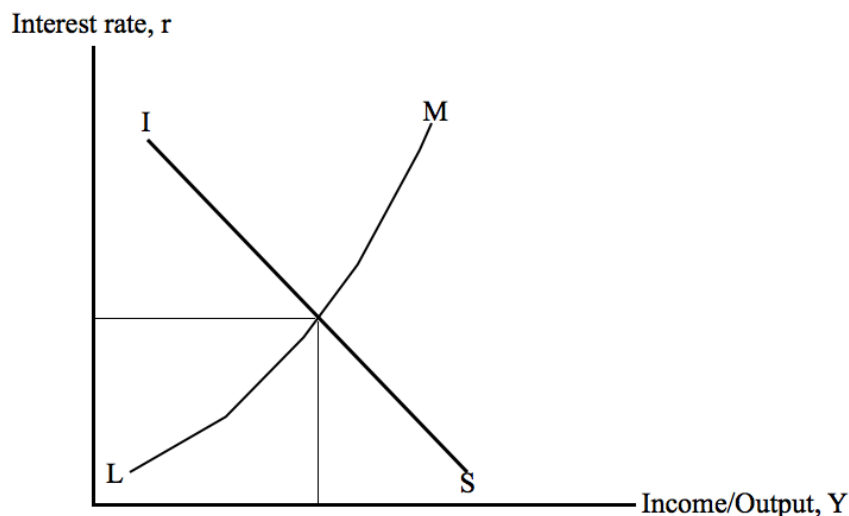


Figure 3: Equilibrium in the IS-LM model

Exercise 1 (Bush’s fiscal stimulus) Consider a standard IS-LM model in equilibrium. Graphically analyse the effects of a large increase in government expenditure financed through taxation on output/income and the interest rate, and briefly explain your reasoning.

Exercise 2 (The credit crunch) Consider a standard IS-LM model in equilibrium. Graphically analyse the effects of a large decrease in the supply of money on output/income and the interest rate, and briefly explain your reasoning.

Exercise 3 (Numerical example) Imagine a closed economy with equilibrium output given by $Y = C + I + G$. Total supply is given by $Y = 5,000$. Consumption is determined by $C = 250 + 0.75(Y - T)$. Investment is given by $I = 1000 - 50r$. Initially, fix G and T at $G = 1,000, T = 1,000$. Suppose the government pursues an expansionary policy, driving G from 1000 to 1250. What happens to national savings? Is there a deficit? How much of one? Will the interest rate decrease or increase? By how much?

Taken together, the IS and LM curves determine the level of aggregate demand (AD) in the economy in the short run. We can also easily verify that the AD curve is downward sloping.

The next condition to verify is the aggregate supply condition.

3 Aggregate supply

Aggregate output in our economy is believed to be determined by a production function, which encapsulates the idea that the economy actually produces things by combining the factors of production—land, labour (L), and capital, K to produce the output of the economy, Y .

This aggregate production function relates the labour input L and the level of the capital stock employed to the level of output in the economy.

$$Y = F(K, L). \quad (23)$$

The time horizon the standard macro model works on is sufficiently short to assume the stock of capital is fixed at some $K = \bar{K}$, leaving labour input, L , as the only variable input of production. The equation for

nominal profit Σ will then be total output minus the wage bill, wL , at a given price level, P .

$$\Sigma = Pf(L^D) - wL. \quad (24)$$

Equation 24 relates nominal profit to the wage rate and the demand for labour, L^D . The expression for aggregate demand for labour in the economy will be given by

$$\frac{W}{P} = \theta L^D, \quad (25)$$

with the assumption of diminishing marginal productivity of labour implying $d\theta/dw < 0$.

Households are assumed to maximise utility over their lifetimes with regard to a work/leisure choice. The household maximises income, Y , and leisure, Le , via

$$\max(Y, Le) \quad (26)$$

subject to

$$Le = Tot - L^S, \quad (27)$$

$$Y = \frac{W}{P} L^S. \quad (28)$$

The household maximises a concave utility function subject to a time constraint given by equation 27 and a supply constraint given by 28. The total number of hours available to work is given by Tot . Maximising this constrained optimisation gives an optimality result where the labour supply function is given by

$$\frac{W}{P} = \phi(L^S) \quad (29)$$

Equilibrium will hold in a labour market where

$$L^S = L^D = L. \quad (30)$$

So, we can write our equilibrium condition in the form

$$\frac{W}{P} = \theta(L) = \phi(L). \quad (31)$$

Solving equation 31 with the production function, the labour market implies an aggregate supply function which is independent of P . Output remains fixed at a full-employment level.

Exercise 4 (Lifetime earnings and the budget constraint) *Imagine a person, let's call her Jill, who lives for two periods. You can think of the periods as episodes, or eras of her life, if you like, such as working and retirement. She earns 200 in period 1, and 50 in period two. Let's assume a credit market exists, and because we are nice people, the credit market is free, that is, there is no interest charged to Jill if she decides to save some of her money. Jill is a smart person, and wants to consume the same amount throughout her life. Without access to a credit market, Jill's consumption stream is $\{200, 50\}$. With a credit market, Jill can consume $200+50/2 = 125$ per period, so her consumption stream at $\{c_1, c_2\}$, which is $\{125, 125\}$. In reality, Jill would buy a bond or a treasury bill to achieve consumption patterns like this. What would her consumption set look like? Draw the level of consumption in period 1 on the y axis, and the level of consumption for period 2 on the x-axis. Comment on the differing levels of utility Jill would experience. What does this tell you about the functions of a money market?*

4 The Phillips curve

The Phillips curve⁴ relates changes in inflation to changes in unemployment. The shape of the curve is given by

$$\dot{p} = \frac{\dot{P}}{P} = \alpha(Y - \bar{Y}) + \pi \quad (32)$$

Here \bar{Y} is the natural(full) employment level of output, and \dot{P}/P is the inflation rate.

Taken together with the IS-LM equations, the IS-LM-Phillips equations define the short run equilibrium value of output in the macroeconomy.

Exercise 5 (The Phillips curve) *Go online, and get a diagram of the Phillips curve. Now draw it for yourself. Leave space for a diagram below the Phillips curve diagram. Now draw an aggregate supply curve. How are the two curves related? Why are they related that way? Give an intuitive explanation. Extra credit: give an algebraic formulation of the relationship.*

5 Dynamics of asset accumulation

This is a lecture about macrodynamics, where the evolution of an economy over time can be studied. Now we've set up our little model, we can look at how best to represent the accumulation of wealth in the economy.

Getting back to the dynamics of asset accumulation, let's look at the budget constraint in each time period.

The budget constraint, if we allow the price of bonds to equal 1, is given by

$$\dot{M} + \dot{B} = P[G - T] + rB. \quad (33)$$

The right hand side of equation 33 gives the government's nominal budget constraint (deficit or surplus) for each time period. This must equal the nominal value of government expenditures of goods and services through out the period. The choice of financial mix (bonds versus money financing) is an important policy decision, and has been well studied in the literature.

The other important source of asset accumulation is investment. This is described by the relationship

$$\dot{K} = I. \quad (34)$$

The study of the dynamics of equation 34 is the province of growth theory, which we won't go into here. A good guide to growth theory is [Barro and Sala-i Martin \(2003\)](#).

5.1 Expectations

Finally, we close our model with an equation concerning the formation of expectations. Here we only concern ourselves with inflationary expectations. This formulation is the adaptive expectations hypothesis, due to [Nerlove \(1958\)](#), and is specified as

$$\dot{\pi} = \gamma(p - \pi). \quad (35)$$

Equation 35 says the rate at which the forecast for inflation ($\dot{\pi}$) is revised at any point in time is proportional to the forecast error already being committed. If the agent's current or most recent forecast of the inflation rate under predicts the corresponding actual inflation rate, the forecast is revised upwards, and vice versa. Mathematically, equation 35 is an exponentially declining weighted average, of which more can be found in [Holt \(2004\)](#).

Under so-called rational expectations assumptions, the rational forecast of inflation is the correct one, so

⁴The canonical study is ([Phillips, 1958](#)).

$$\pi = p. \tag{36}$$

This closes our model.

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